The Hidden Geometry of Electromagnetism

by Yaron Hadad

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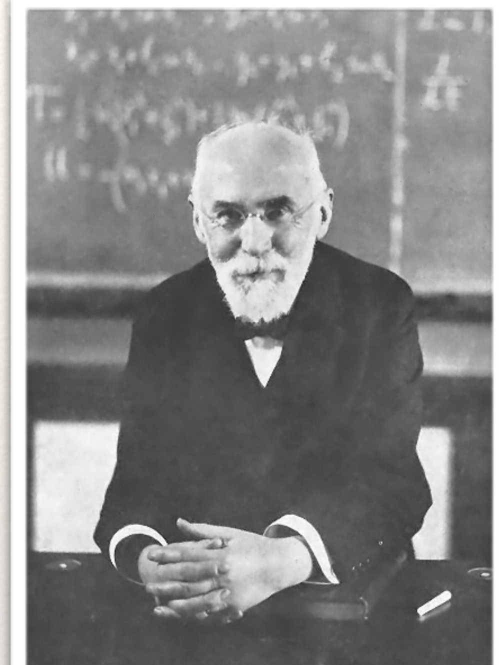
The Problem of Radiation-Reaction

The Lorentz Force (LF) Eq: $m\dot{u}^\alpha = -eF^{\alpha\beta}_{\text{ext}}u_\beta$



Lorentz 1892

The rate at which energy is radiated away from the electron is

$$\mathcal{R} = -m\tau_0\dot{u}^\alpha\dot{u}_\alpha$$

$$\tau_0 = \frac{2}{3}\frac{e^2}{m} = 6.24 \times 10^{-24}\,\text{s}$$

$\Longrightarrow$ an accelerating charge loses energy.

This effect is not included in the Lorentz Force equation. The rate at which energy-momentum is emitted by radiation:

$$\frac{dP^\alpha}{d\tau} = \mathcal{R}u^\alpha$$

The Problem of Self-Force

Problem #1: Dynamics of a particle in a known external field

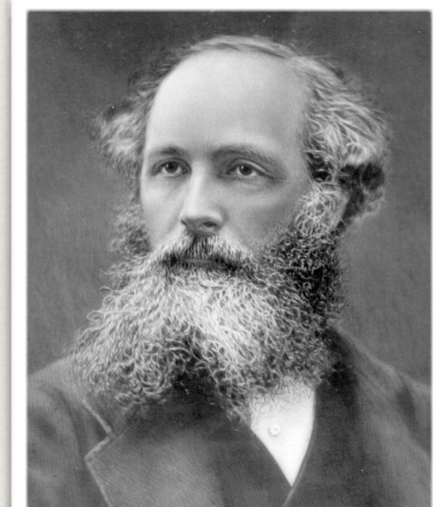
$m\dot{u}^\alpha = -eF^{\alpha\beta}_{\rm ext} u_\beta$ well-posed for *any* external field $F^{\alpha\beta}_{\rm ext}$

Problem #2: Dynamics of the field for known currents

$\partial_\alpha F^{\alpha\beta} = 4\pi J^\beta$ well-posed if $\partial_\alpha J^\alpha = 0$

Also well-defined for a point particle

$$J^\alpha(x) = -eu^\alpha \delta(x - z(t)) \frac{d\tau}{dt}$$



Maxwell 1865

Problem #3 = #1 + #2: The coupled system

Mathematically ill-defined

A Plentitude of Models…

Schott

radiation-reaction

Lorentz-Abraham-Dirac (193…)

$$m\dot{u}^{\alpha} = -eF^{\alpha\beta}u_{\beta} + m\tau_0\left[\ddot{u}^{\alpha} + \dot{u}^2 u^{\alpha}\right]$$

Landau-Li…

…rce

$\ldots{}^{\beta\gamma}F_{\gamma\delta}u^{\delta}u_{\beta}u^{\alpha})$

$\ldots\dot{u}_{\gamma}u^{\alpha}\,]$

$\ldots{}^{\alpha}\,]$

$\ldots\tau)$

A Rigorous Derivation of Electromagnetic Self-force

Samuel E. Gralla, Abraham I. Harte, and Robert M. Wald

Effects of Radiation-Reaction in Relativistic Laser Acceleration

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perturbative equations. … moment,

the Landau-Lifshitz equation is solved analytically for an arbitrary (transverse) electromagnetic pulse. A comparative study of the radiation emission of an electron in a linearly polarized pulse

More equations: Prigogine-Henin (1962), Nodvik (1964), Teitelboim (1970), Gonzales-Gascon (1976), Petzold-Sorg (1977), Ford-O'Connell (1991), Sokolov et al. (2009), Hammond (2011), Cabo-Castineiras (2013), more?

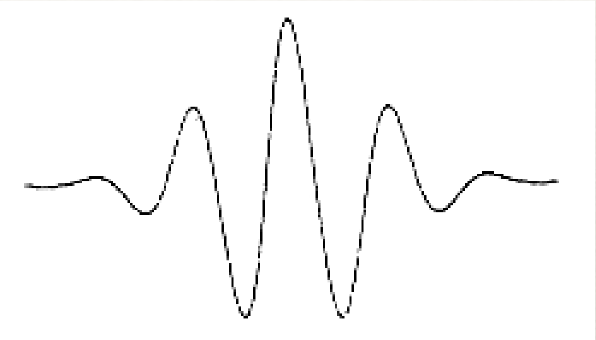
Setup

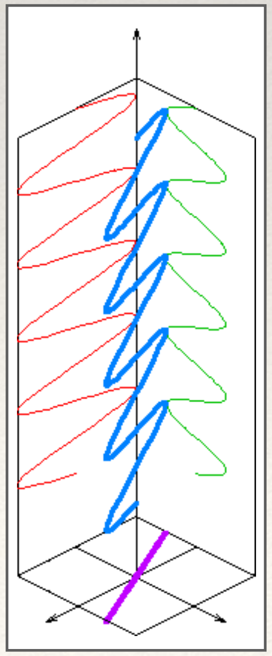
Transverse wave: $A^\alpha(x) = A_0 \mathrm{Re}\left[\varepsilon^\alpha f\left(\xi\right)\right]$ $\hat{A}^\alpha = A^\alpha / A_0$

- Polarization vector ε^α
- Wave vector k^α
- Phase $\xi = k \cdot x = \omega t - \vec{k} \cdot \vec{x}$

Transverse condition: $k \cdot \varepsilon = 0$

Can still be a pulse:



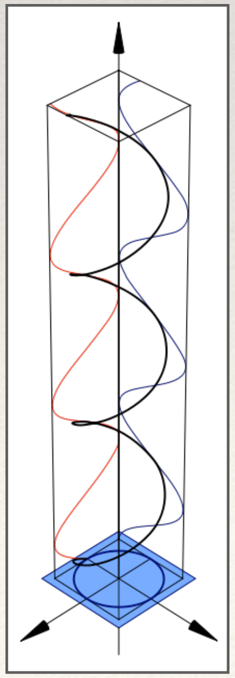


Linear polarization

$$\begin{aligned} \varepsilon^\alpha &= (0,0,1,0) \\ k^\alpha &= (\omega, k, 0, 0) \\ f(\xi) &= A_0 \cos(\xi) \\ \vec{A} &= A_0 \cos(\xi) \hat{y} \end{aligned}$$

Circular polarization

$$\begin{aligned} \varepsilon^\alpha &= \frac{1}{\sqrt{2}}(0,0,1,\pm i) \\ k^\alpha &= (\omega, k, 0, 0) \\ f(\xi) &= A_0 e^{i\xi} \\ \vec{A} &= \frac{A_0}{\sqrt{2}}\left[\cos(\xi)\hat{y} \mp \sin(\xi)\hat{z}\right] \end{aligned}$$



Radiation-Reaction Dominated Regime (RRDR)

The rate at which energy is radiated away from the electron is:

$$\mathcal{R} = -\frac{2}{3}e^2\frac{(k\cdot u)^4}{(k\cdot u_0)^2}\{a_0^2\hat{A}'^2$$

Lorentz

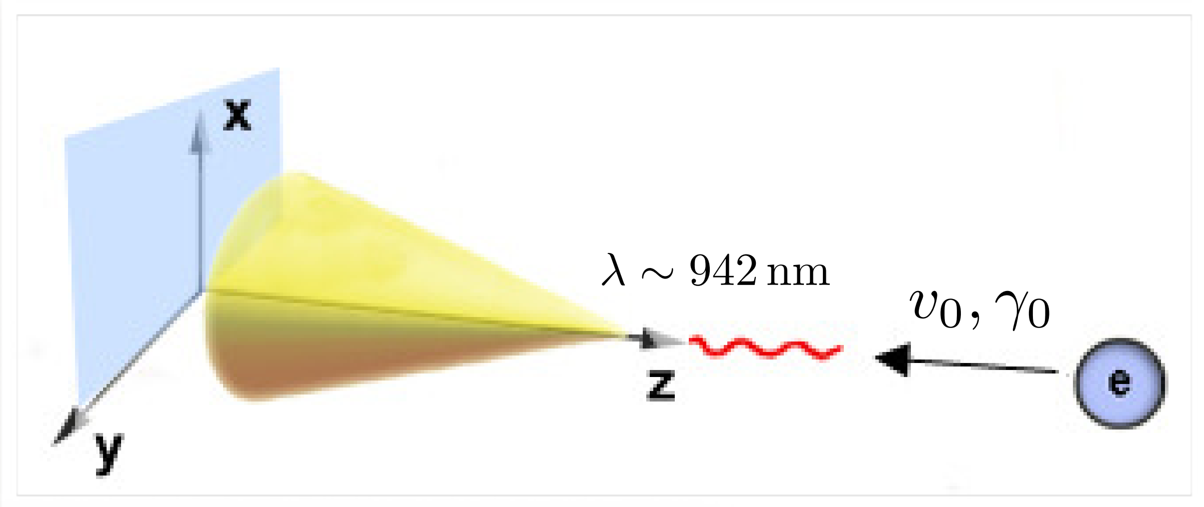
$$a_0 = \frac{eA_0}{m}$$

$$+2(k\cdot u_0)\tau_0\left[a_0^2\hat{A}'\cdot\hat{A}'' - a_0^4\Psi\hat{A}'^2\right] + O(\tau_0^2)\}$$

Landau-Lifshitz (RR) correction $\sim (k\cdot u_0)\tau_0 a_0^4$

$$k\cdot u_0 = \gamma_0(\omega - \vec{k}\cdot\vec{v_0})$$

Radiation-reaction is important if $a_0^2 \sim (\omega\tau_0)\gamma_0 a_0^4$



RRDR criterion: $\gamma_0 a_0^2 \sim 10^8$

Probing Radiation-Reaction

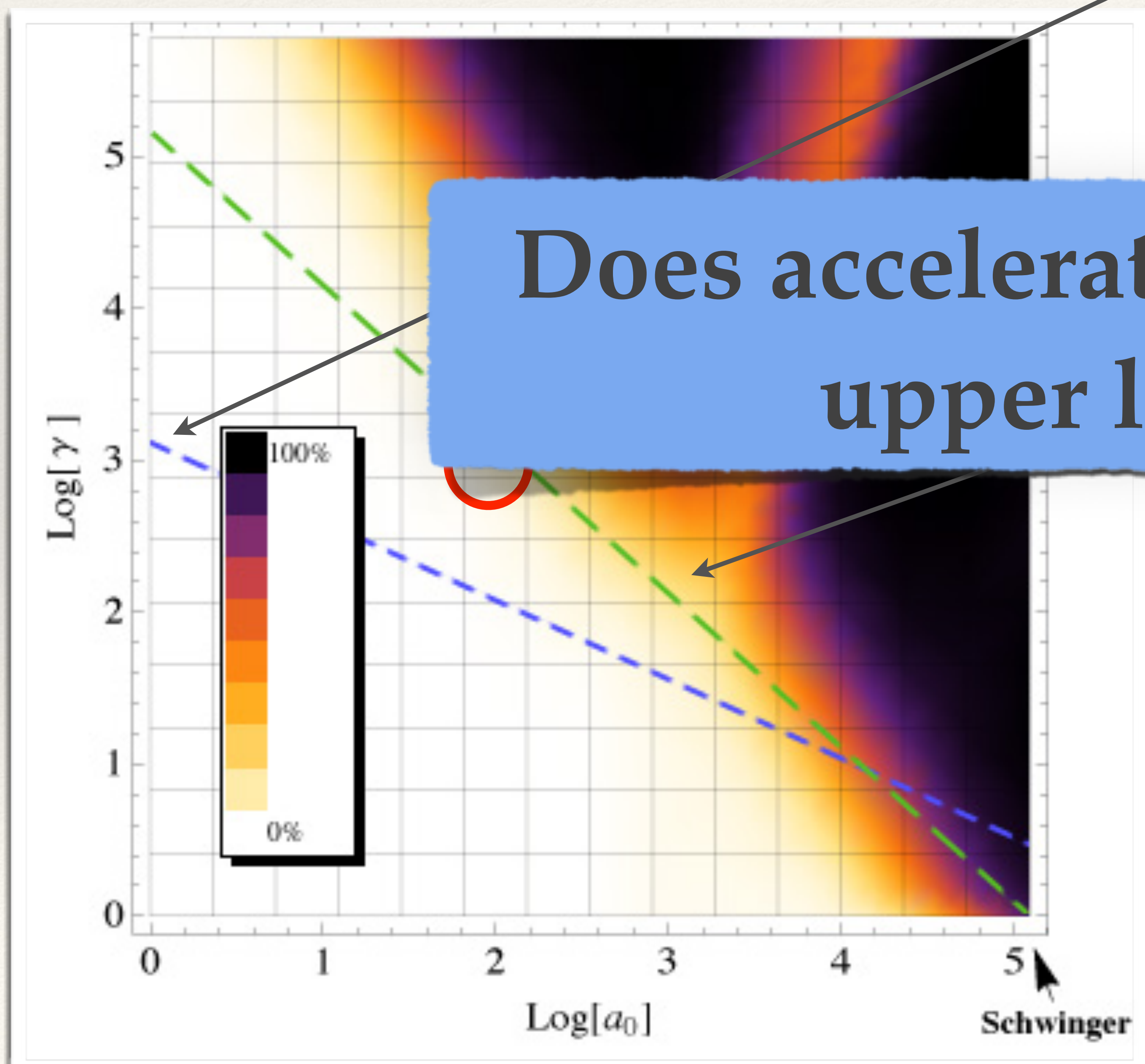
$$\Delta = \frac{1}{2\pi}\int_0^{2\pi} \frac{|E_{LL}(\xi) - E_{\text{Lorentz}}(\xi)|}{|E_{LL}(\xi) + E_{\text{Lorentz}}(\xi)|} d\xi$$

Different LL and LAD dynamics

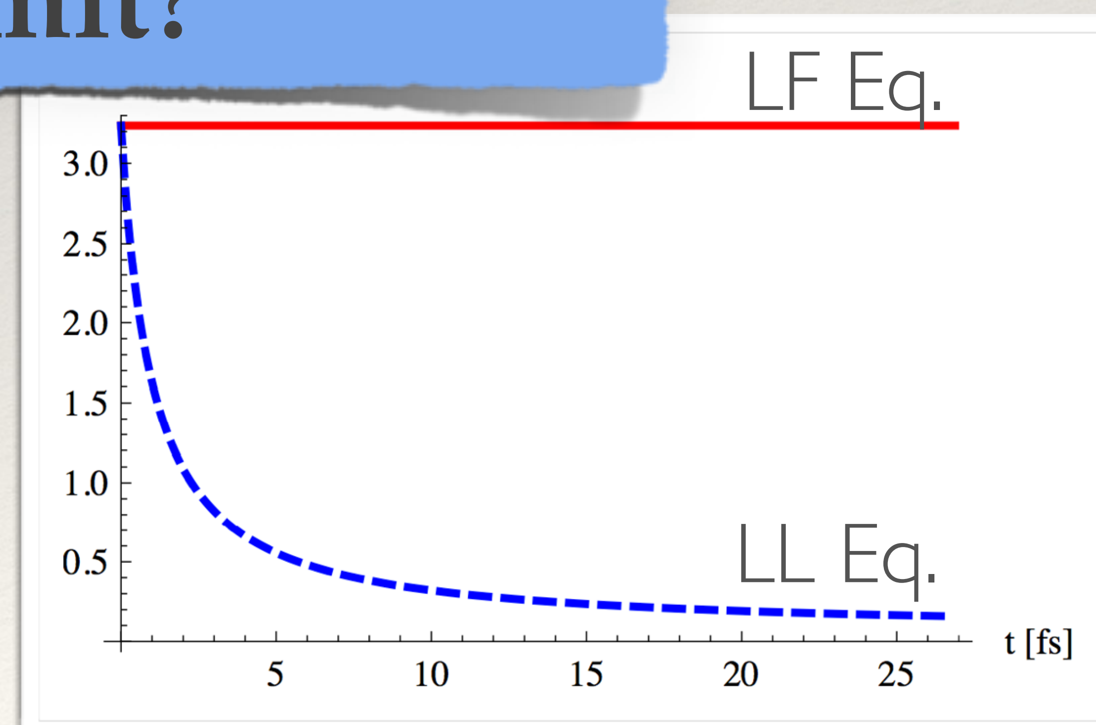
Critical acceleration

$$a_c = \frac{mc^3}{\hbar}$$

$$a_c = 2.3 \times 10^{29} \, \frac{m}{s^2}$$



Does acceleration have an upper limit?

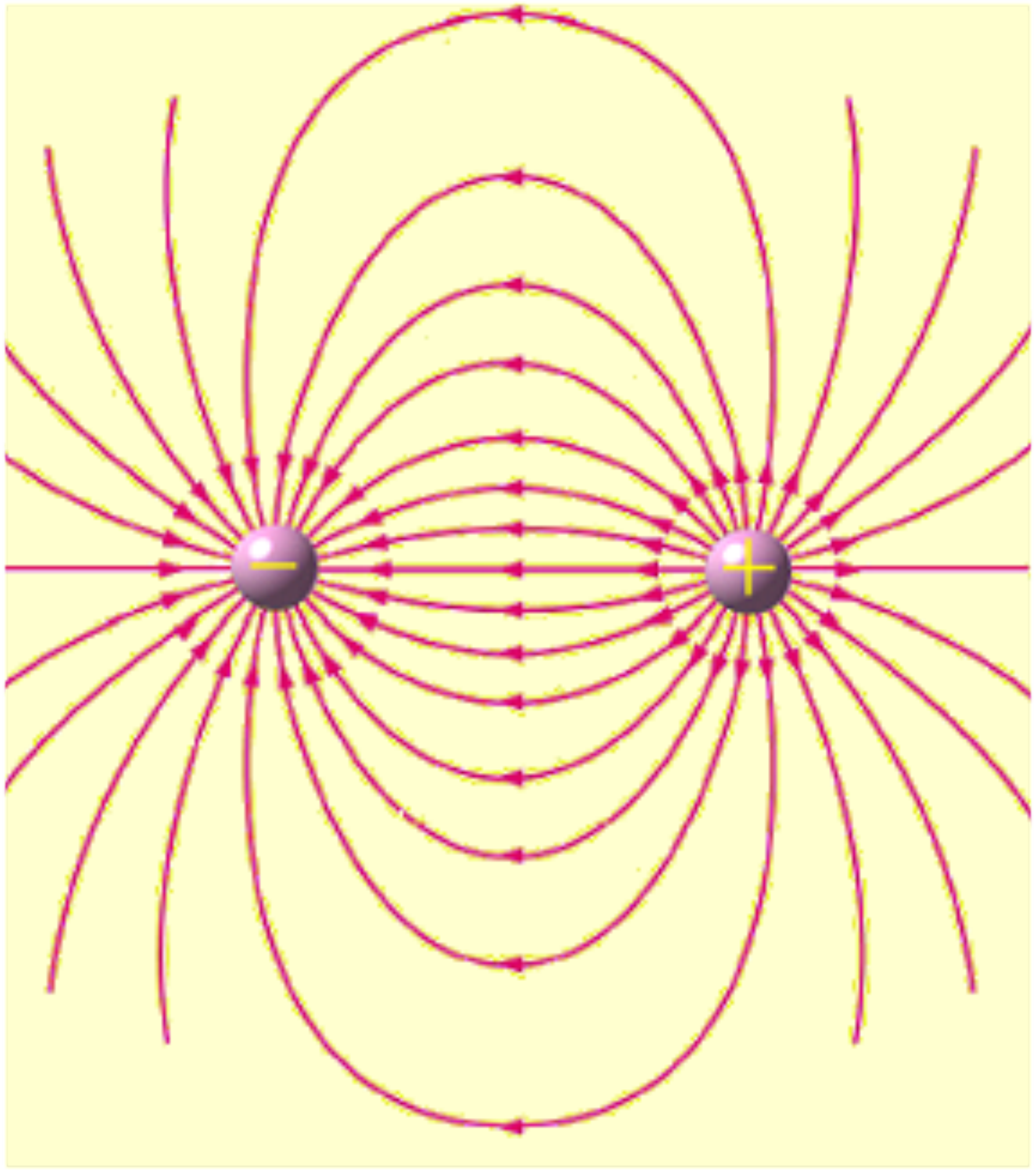


CP Wave $E_e = 0.511\, GeV$

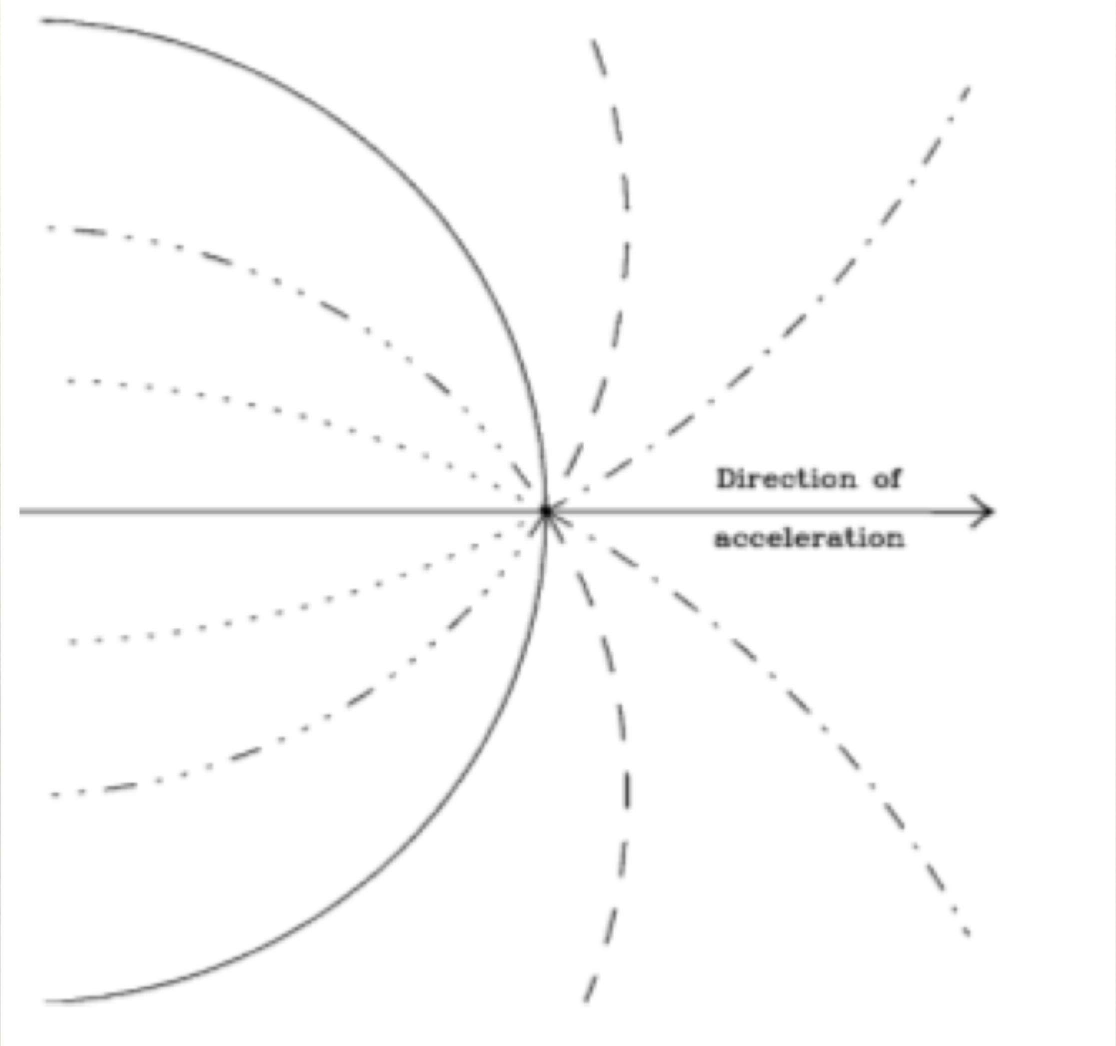
Part II

Field Lines

Michael Faraday



Field Lines Curvature



Charge acceleration $\Longleftrightarrow$ Field lines curvature

Electric Curvature

The curvature of a curve $\vec{\gamma}$

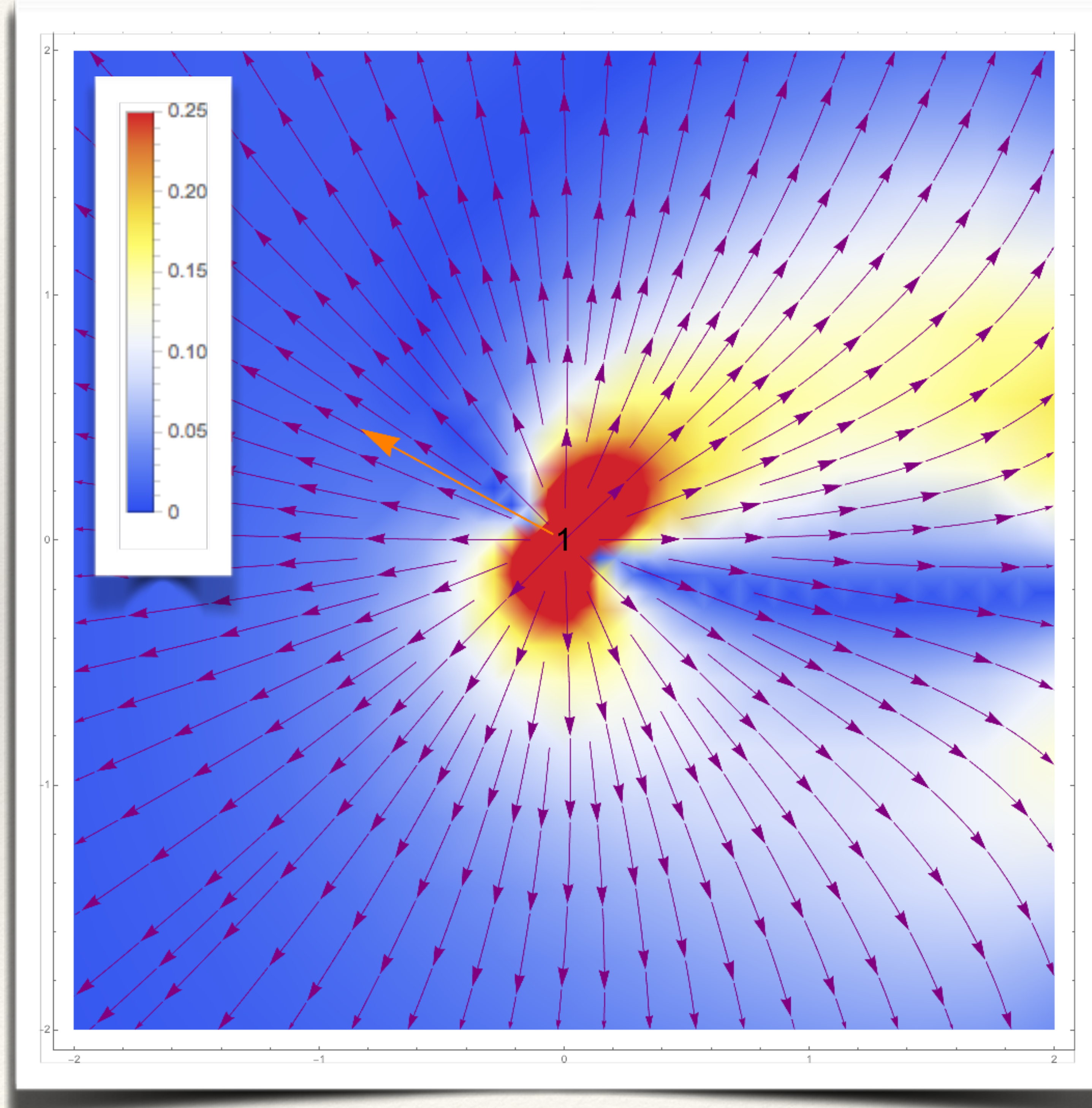
$$k = \frac{|\vec{\gamma}' \times \vec{\gamma}''|}{|\vec{\gamma}'|}$$

For an electric field line:

$$k = \frac{|\vec{E} \times (\vec{E} \cdot \nabla)\vec{E}|}{|\vec{E}|^3}$$

$$\vec{E}(\vec{x}) = \underbrace{\vec{E}_{\text{self}}(\vec{x})} + \vec{E}_{\text{ext}}(\vec{x})$$

Coulomb's law in the electron's rest frame



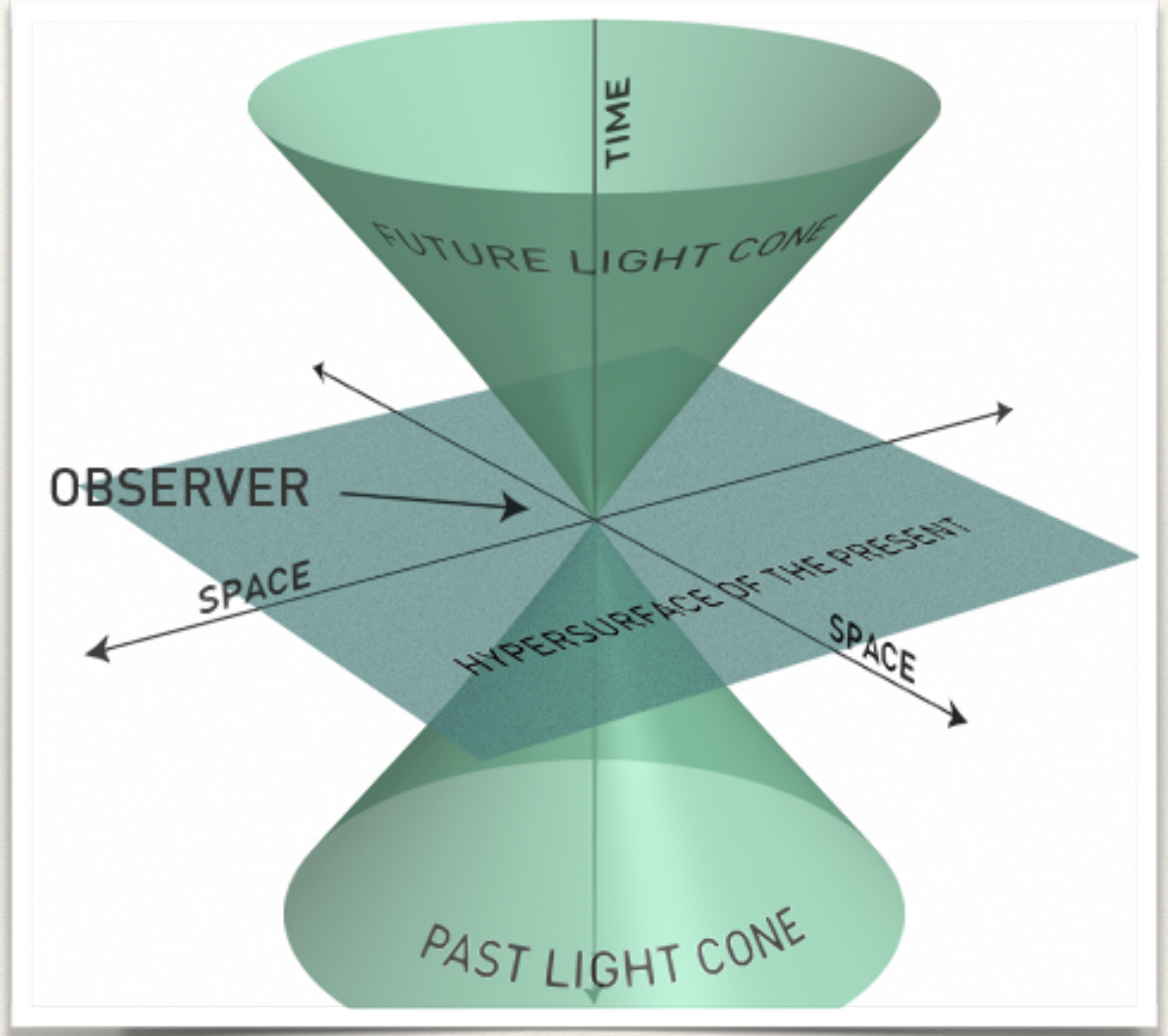
Electromagnetic Geodesics

$$k(\vec{x}) \approx \frac{3}{q}|\underbrace{\vec{E}_{\text{ext}}(\vec{x}) \times (\vec{x} - \vec{x}_0(t))}|$$

Never singular, not even for point-like particles

$$k(\vec{x}_0(t+\Delta t)) \approx \frac{3(\Delta t)^2}{2q}|\vec{E}_{\text{ext}} \times \vec{a}_0(t)|$$

$= 0 \Longleftrightarrow$ The charge accelerates along an **"electromagnetic geodesic"** (lines of zero curvature)



Charges accelerate along **trajectories of least electromagnetic curvature**

Thank you!

read more at

www.yaronhadad.com