Electrodynamics in the high-acceleration regime

by Yaron Hadad

Credits to:

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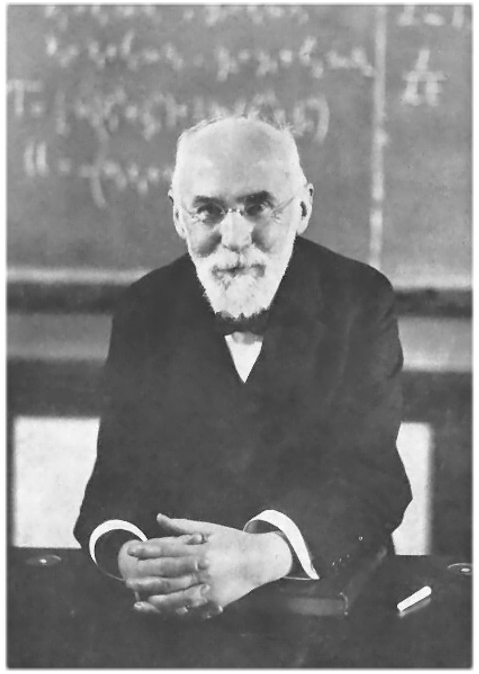
The Problem of Radiation-Reaction

The Lorentz Force (LF) Eq: $m\dot{u}^\alpha = -eF^{\alpha\beta}_{\text{ext}}u_\beta$

The rate at which energy is radiated away from the electron is

$$\mathcal{R} = -m\tau_0 \dot{u}^\alpha \dot{u}_\alpha$$

$$\tau_0 = \frac{2}{3}\frac{e^2}{m} = 6.24 \times 10^{-24}\,\text{s}$$



Lorentz 1892

$\Longrightarrow$ an accelerating charge loses energy.

This effect is not included in the Lorentz Force equation. The rate at which energy-momentum is emitted by radiation:

$$\frac{dP^\alpha}{d\tau} = \mathcal{R}u^\alpha$$

The Problem of Self-Force

Problem #1: Dynamics of a particle in a known external field

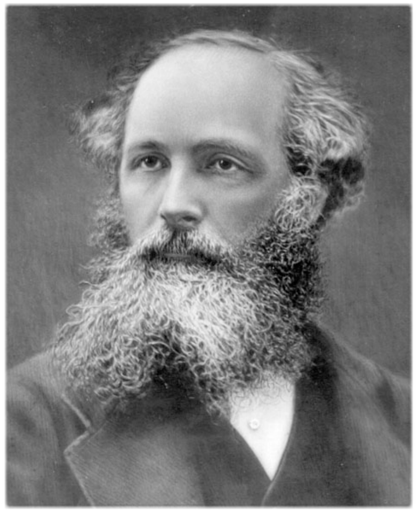
$m\dot{u}^\alpha = -eF^{\alpha\beta}_{\text{ext}} u_\beta$ well-posed for *any* external field $F^{\alpha\beta}_{\text{ext}}$

Problem #2: Dynamics of the field for known currents

$\partial_\alpha F^{\alpha\beta} = 4\pi J^\beta$ well-posed if $\partial_\alpha J^\alpha = 0$

Also well-defined for a point particle

$$J^\alpha(x) = -eu^\alpha \delta(x - z(t)) \frac{d\tau}{dt}$$



Maxwell 1865

Problem #3 = #1 + #2: The coupled system

Mathematically ill-defined

A Plentitude of Models…

Schott

radiation-reaction

Lorentz-Abraham-Dirac (1938)	$m\dot{u}^\alpha = -eF^{\alpha\beta}u_\beta + m\tau_0\left[\ddot{u}^\alpha + \dot{u}^2 u^\alpha\right]$
Landau-Li…	…force …$^{\beta\gamma}F_{\gamma\delta}u^\delta u_\beta u^\alpha)$
Elieze…	…$^{\beta\gamma}u_\beta \dot{u}_\gamma u^\alpha\big]$
	…$^\alpha\big]$
	…$\tau)$
	…$)\big]$

A Rigorous Derivation of Electromagnetic Self-force

Samuel E. Gralla, Abraham I. Harte, and Robert M. Wald

Enrico Fermi Institute and Department of Physics

University of Chicago

magnetism. We considered a one-parameter-family of solutions to the Maxwell and matter equations (1)-(3) and (7) containing a body that "shrinks down" to zero size, mass, and charge according to the scaling assumptions of section II. We found that the lowest-order

perturbative equations. In the case of negligible spin and electromagnetic dipole moment, this reduces to the reduced-order ALD equation.

More equations: Prigogine-Henin (1962), Nodvik (1964), Teitelboim (1970), Gonzales-Gascon (1976), Petzold-Sorg (1977), Ford-O'Connell (1991), Sokolov et al. (2009), Hammond (2011), Cabo-Castineiras (2013), more?

What Does QED Say?

Lorentz-A

(

$+ \dot{u}^2 u^\alpha]$

Radiation reaction from QED: lightfront perturbation theory in a plane wave background

The Lorentz-Dirac force from QED for linear acceleration

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(Dated: July 20, 2004)

We investigate the motion of a wave packet of a charged scalar particle linearly accelerated by a static potential in quantum electrodynamics. We calculate the expectation value of the position of the charged particle after the acceleration to first order in the fine structure constant in the $\hbar \to 0$ limit. We find that the change in the expectation value of the position (the position shift) due to radiation reaction agrees exactly with the result obtained using the Lorentz-Dirac force in classical electrodynamics. We also point out that the one-loop correction to the potential may contribute to

consistent with QED. This is consistent with known first-order relations between these three equations [8, 9, 27], and with previous results on the derivation of RR from QED [29, 57].

Sokolov et al. (2009), Hammond (2011), Cabo-Castineiras (2013), more?

What Does QED Say?

Lorentz-Abraham-Dirac (1938)	$m\dot{u}^{\alpha} = -eF^{\alpha\beta}u_{\beta} + m\tau_0 \left[\ddot{u}^{\alpha} + \dot{u}^2 u^{\alpha}\right]$
Landau-Lifshitz (1952)	$m\dot{u}^{\alpha} = -eF^{\alpha\beta}u_{\beta} - e\tau_0 \left[F^{\alpha\beta}_{,\gamma}u_{\beta}u^{\gamma} - \frac{e}{m}\left(F^{\alpha\beta}F_{\beta\gamma}u^{\gamma} - F^{\beta\gamma}F_{\gamma\delta}u^{\delta}u_{\beta}u^{\alpha}\right)\right]$
Eliezer (1948)	$m\dot{u}^{\alpha} = -eF^{\alpha\beta}u_{\beta} - e\tau_0 \left[\frac{d}{d\tau}(F^{\alpha\beta}u_{\beta}) - F^{\beta\gamma}u_{\beta}\dot{u}_{\gamma}u^{\alpha}\right]$
~~Mo-Papas (1971)~~	$m\dot{u}^{\alpha} = -eF^{\alpha\beta}u_{\beta} - e\tau_0 \left[F^{\alpha\beta}\dot{u}_{\beta} + F^{\beta\gamma}\dot{u}_{\beta}u_{\gamma}u^{\alpha}\right]$
Caldirola (1979)	$\frac{m}{\tau_0}\left[u^{\alpha}(\tau-\tau_0) - u^{\alpha}(\tau)u_{\beta}(\tau)u^{\beta}(\tau-\tau_0)\right] = -eF^{\alpha\beta}(\tau)u_{\beta}(\tau)$
Caldirola-Yaghjian (1992)	$m\dot{u}^{\alpha} = -eF^{\alpha\beta}(\tau)u_{\beta}(\tau) - \frac{m}{\tau_0}\left[u^{\alpha}(\tau-\tau_0) - u^{\alpha}(\tau)u_{\beta}(\tau)u^{\beta}(\tau-\tau_0)\right]$

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Effects of Radiation-Reaction in Relativistic Laser Acceleration

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(Dated: 14 November, 2010)

the Landau-Lifshitz equation is solved analytically for an arbitrary (transverse) electromagnetic pulse. A comparative study of the radiation emission of an electron in a linearly polarized pulse

More equations: Prigogine-Henin (1962), Nodvik (1964), Teitelboim (1970), Gonzales-Gascon (1976), Petzold-Sorg (1977), Ford-O'Connell (1991), Sokolov et al. (2009), Hammond (2011), Cabo-Castineiras (2013), more?

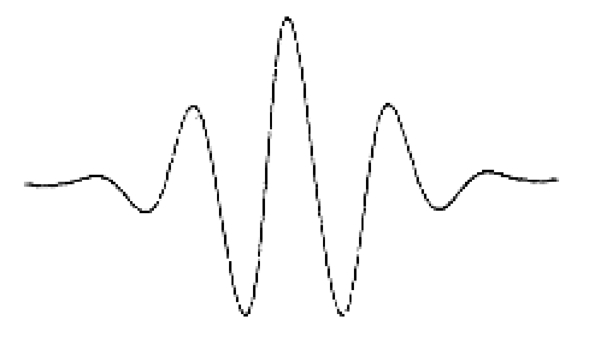
Setup

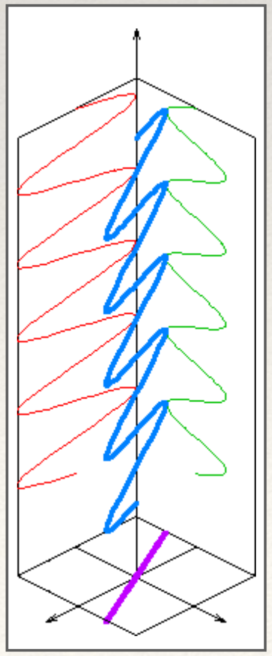
Transverse wave: $A^\alpha(x) = A_0 \mathrm{Re}\left[\varepsilon^\alpha f(\xi)\right]$ $\hat{A}^\alpha = A^\alpha / A_0$

- Polarization vector ε^α
- Wave vector k^α
- Phase $\xi = k \cdot x = \omega t - \vec{k} \cdot \vec{x}$

Transverse condition: $k \cdot \varepsilon = 0$

Can still be a pulse:



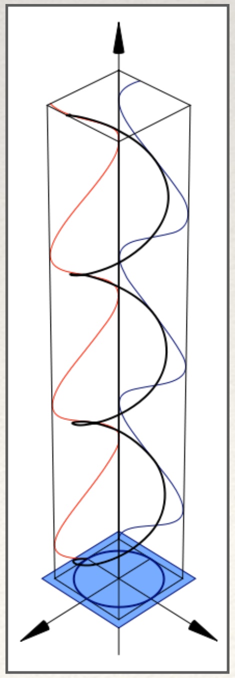


Linear polarization

$$\begin{aligned} \varepsilon^\alpha &= (0,0,1,0) \\ k^\alpha &= (\omega, k, 0, 0) \\ f(\xi) &= A_0 \cos(\xi) \\ \vec{A} &= A_0 \cos(\xi)\hat{y} \end{aligned}$$

Circular polarization

$$\begin{aligned} \varepsilon^\alpha &= \frac{1}{\sqrt{2}}(0,0,1,\pm i) \\ k^\alpha &= (\omega, k, 0, 0) \\ f(\xi) &= A_0 e^{i\xi} \\ \vec{A} &= \frac{A_0}{\sqrt{2}}\left[\cos(\xi)\hat{y} \mp \sin(\xi)\hat{z}\right] \end{aligned}$$



Radiation-Reaction Dominated Regime (RRDR)

The rate at which energy is radiated away from the electron is:

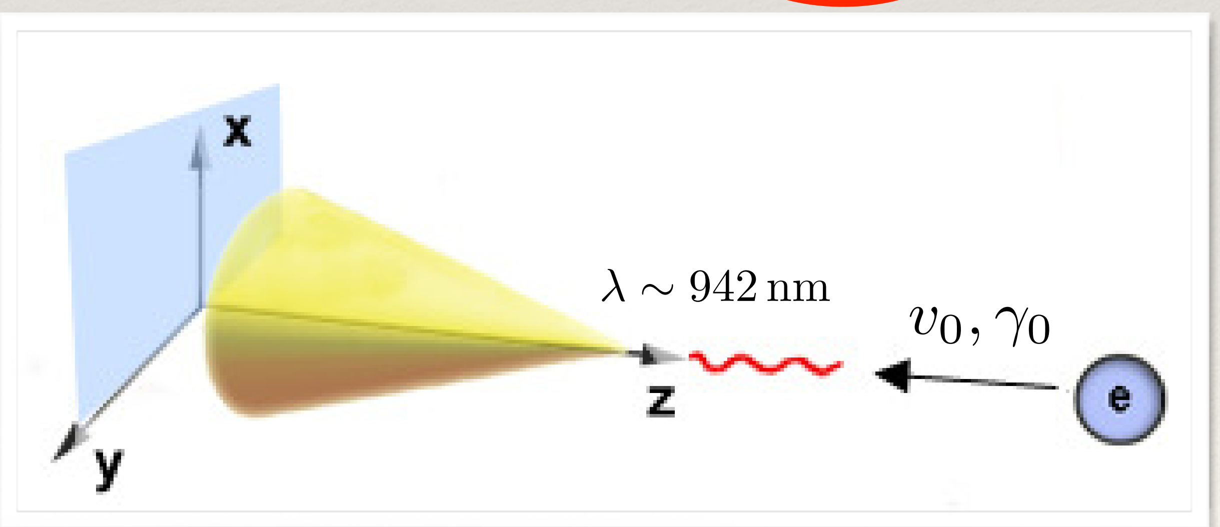
$$\mathcal{R} = -\frac{2}{3}e^2\frac{(k\cdot u)^4}{(k\cdot u_0)^2}\{a_0^2\hat{A}'^2 \quad \text{Lorentz} \qquad a_0 = \frac{eA_0}{m}$$

$$+2(k\cdot u_0)\tau_0\left[a_0^2\hat{A}'\cdot\hat{A}'' - a_0^4\Psi\hat{A}'^2\right] + O(\tau_0^2)\}$$

Landau-Lifshitz (RR) correction $\sim (k\cdot u_0)\tau_0 a_0^4$

$$k\cdot u_0 = \gamma_0(\omega - \vec{k}\cdot\vec{v_0})$$

Radiation-reaction is important if $a_0^2 \sim (\omega\tau_0)\gamma_0 a_0^4$



RRDR criterion: $\gamma_0 a_0^2 \sim 10^8$

Probing Radiation-Reaction

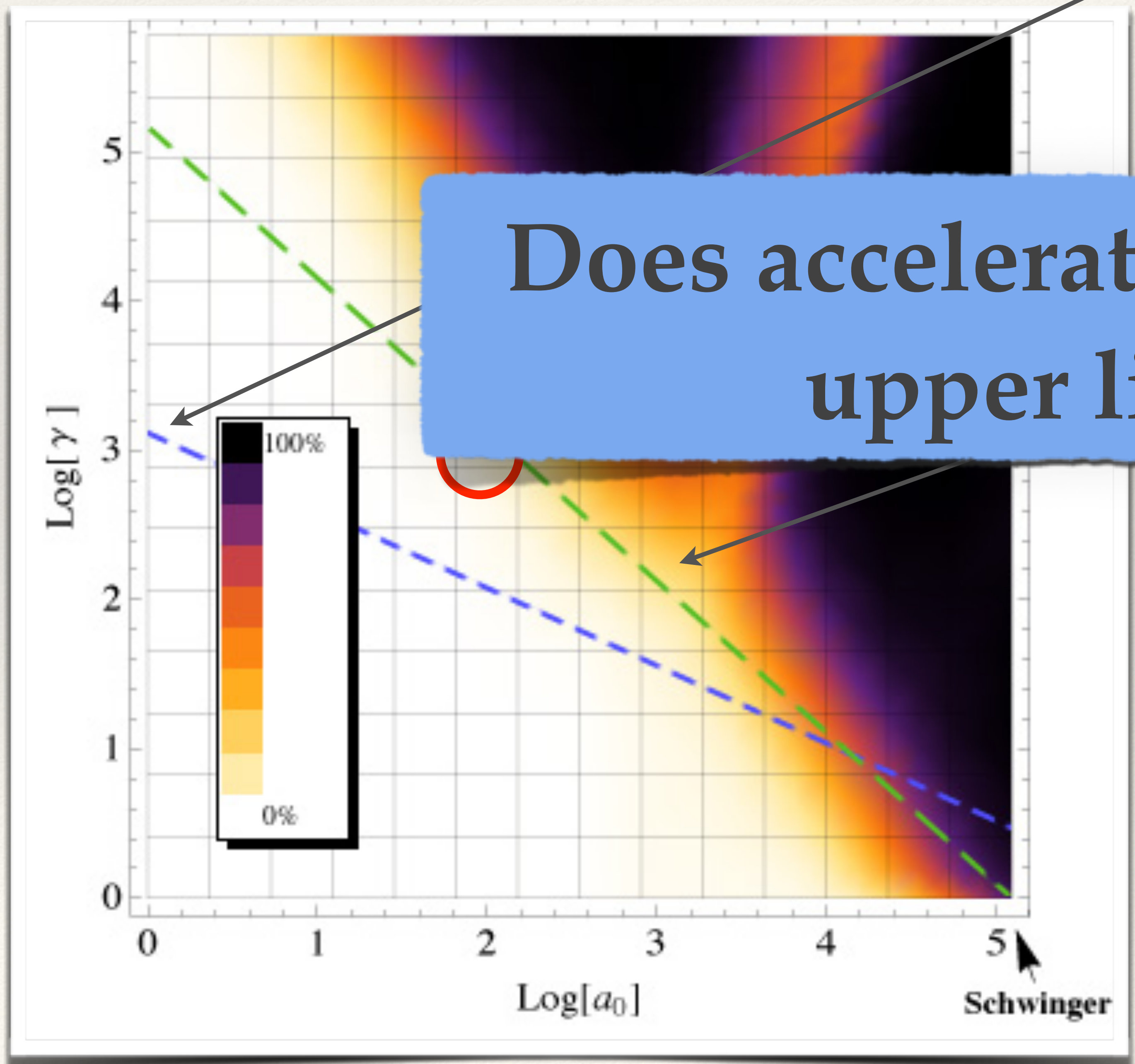
$$\Delta = \frac{1}{2\pi}\int_0^{2\pi} \frac{|E_{LL}(\xi) - E_{\text{Lorentz}}(\xi)|}{|E_{LL}(\xi) + E_{\text{Lorentz}}(\xi)|} d\xi$$

Different LL and LAD dynamics

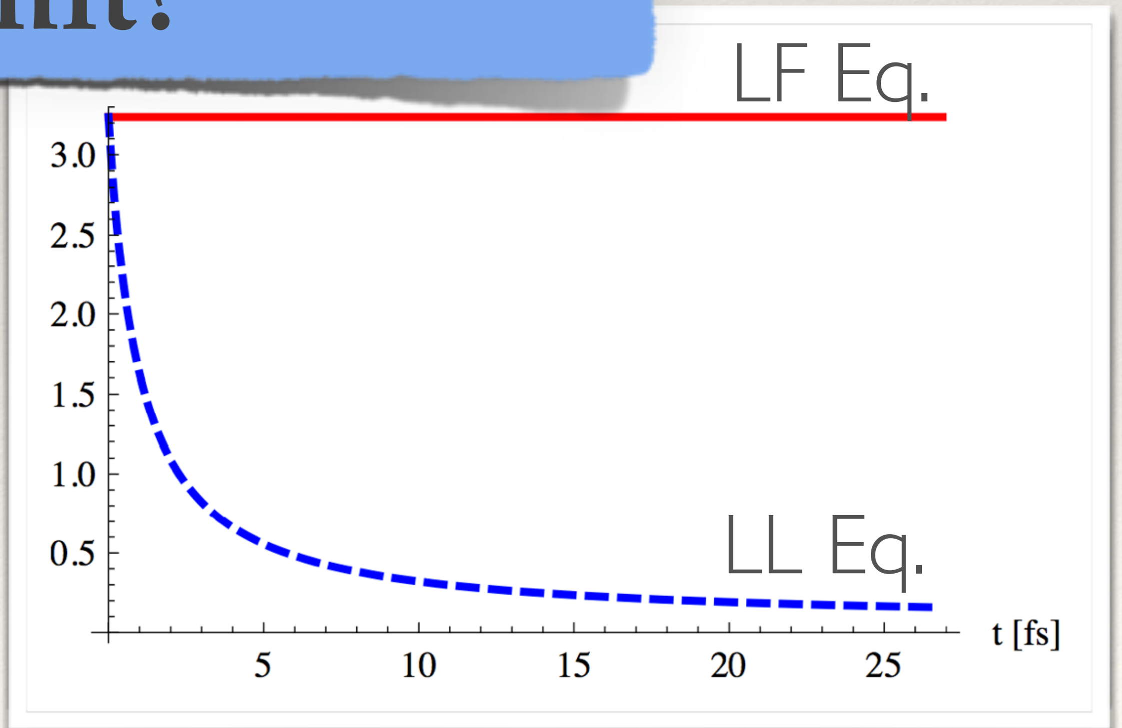
Critical acceleration

$$a_c = \frac{mc^3}{\hbar}$$

$$a_c = 2.3 \times 10^{29}\,\frac{m}{s^2}$$



Does acceleration have an upper limit?

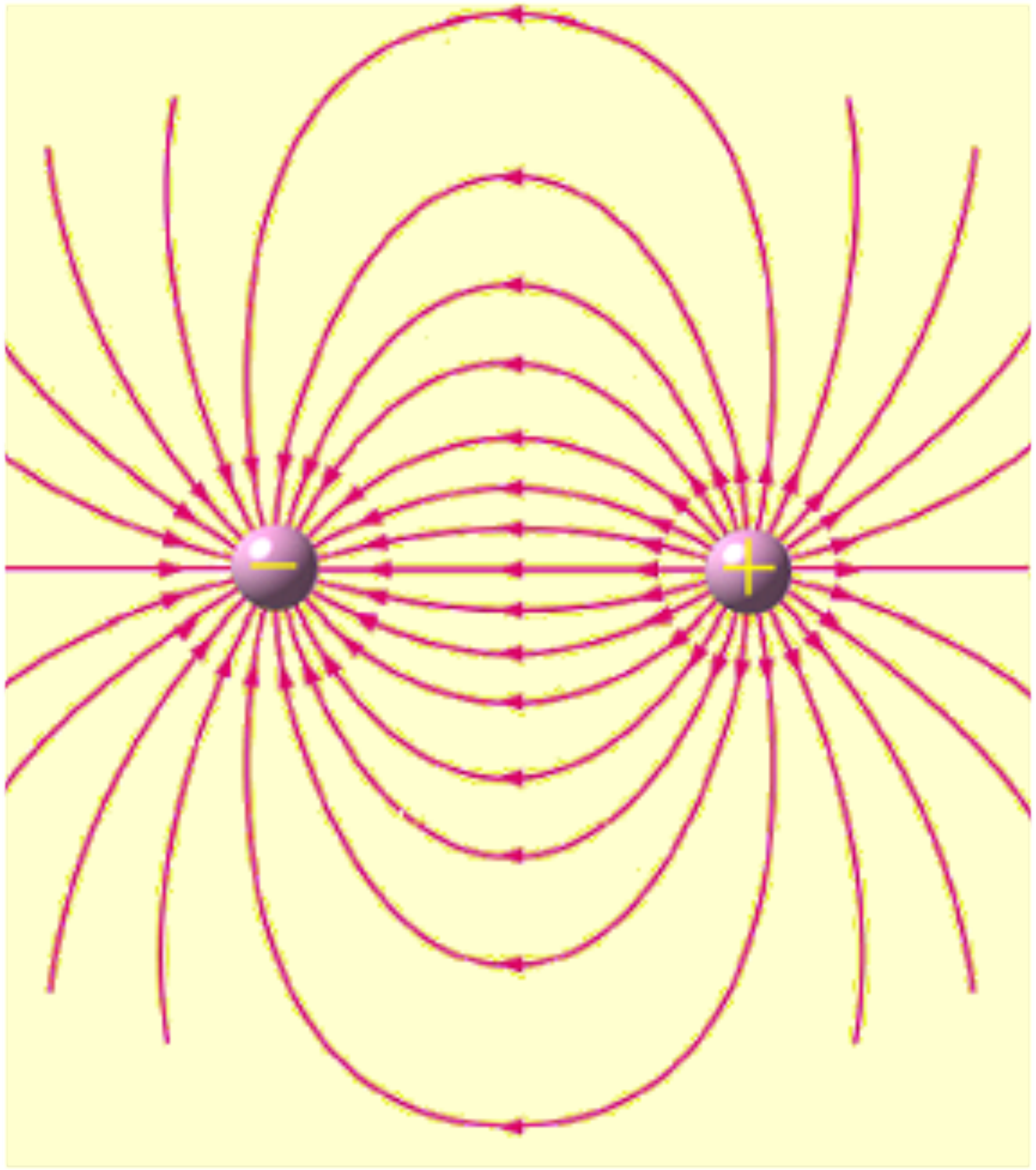


CP Wave $\quad E_e = 0.511\,GeV$

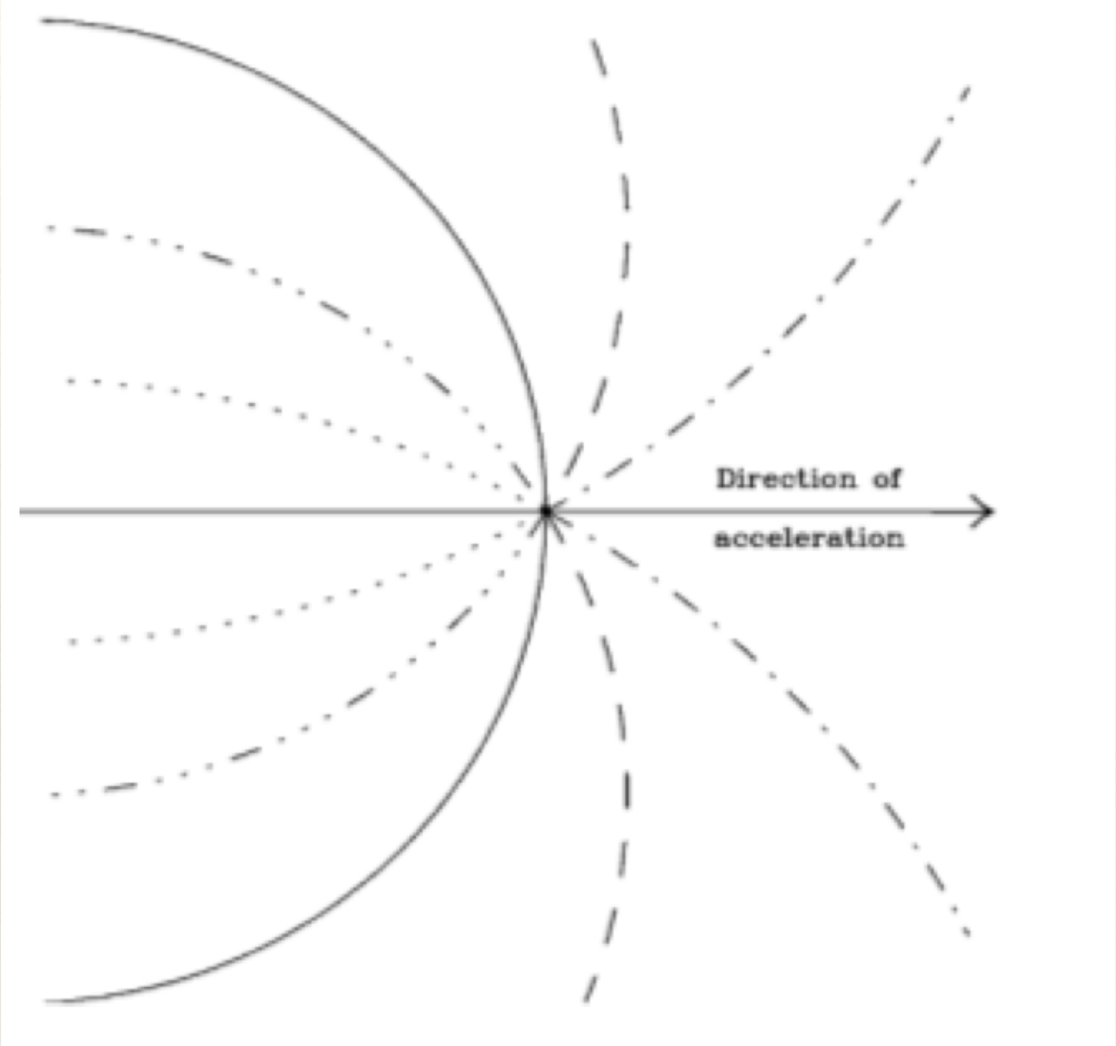
Part II

Field Lines

Michael Faraday



Field Lines Curvature



Charge acceleration $\Longleftarrow\!\!\Longrightarrow$ Field lines curvature

Electric Curvature

The curvature of a curve $\vec{\gamma}$

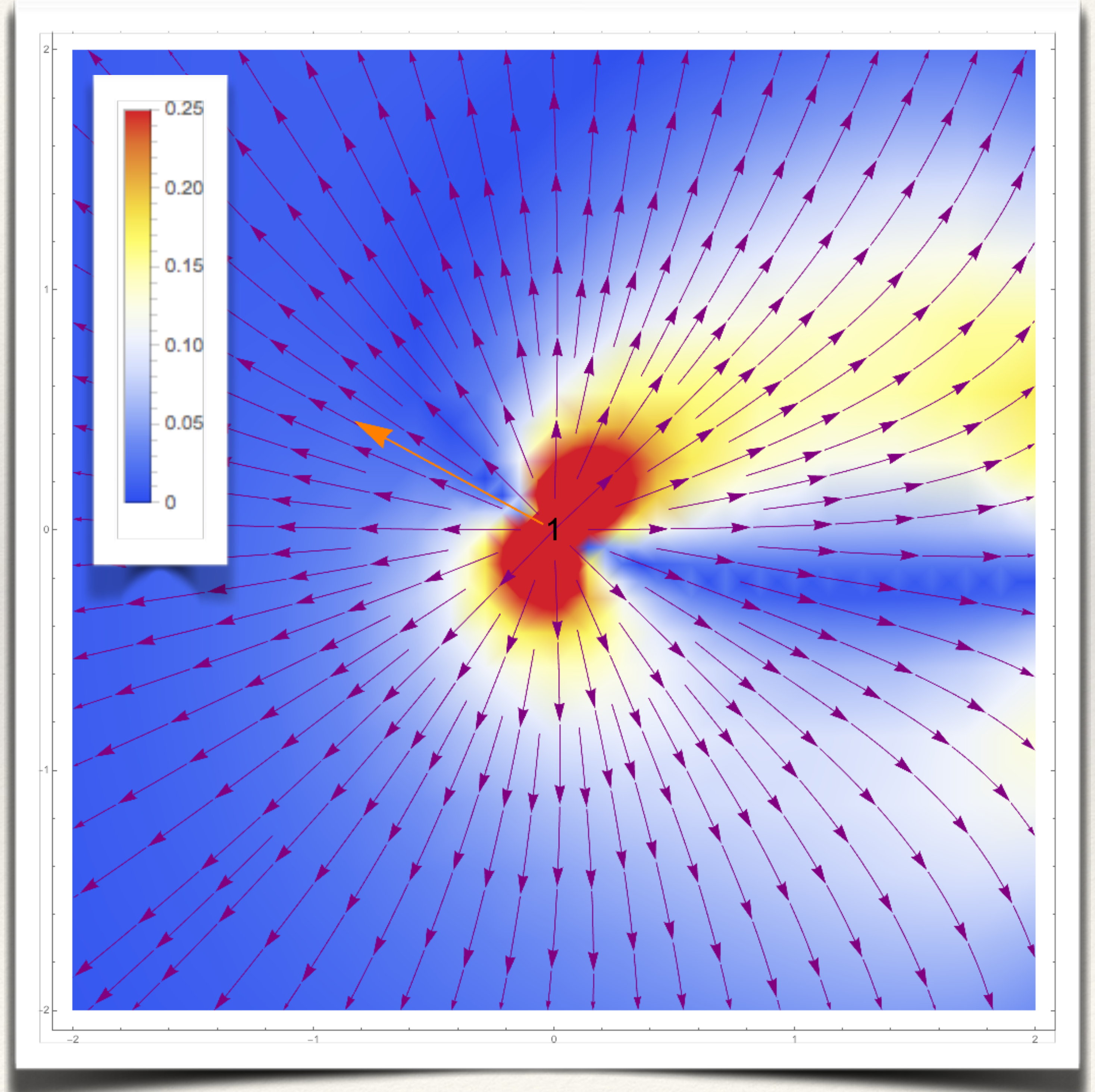
$$k = \frac{|\vec{\gamma}' \times \vec{\gamma}''|}{|\vec{\gamma}'|}$$

For an electric field line:

$$k = \frac{|\vec{E} \times (\vec{E} \cdot \nabla)\vec{E}|}{|\vec{E}|^3}$$

$$\vec{E}(\vec{x}) = \underbrace{\vec{E}_{\text{self}}(\vec{x})} + \vec{E}_{\text{ext}}(\vec{x})$$

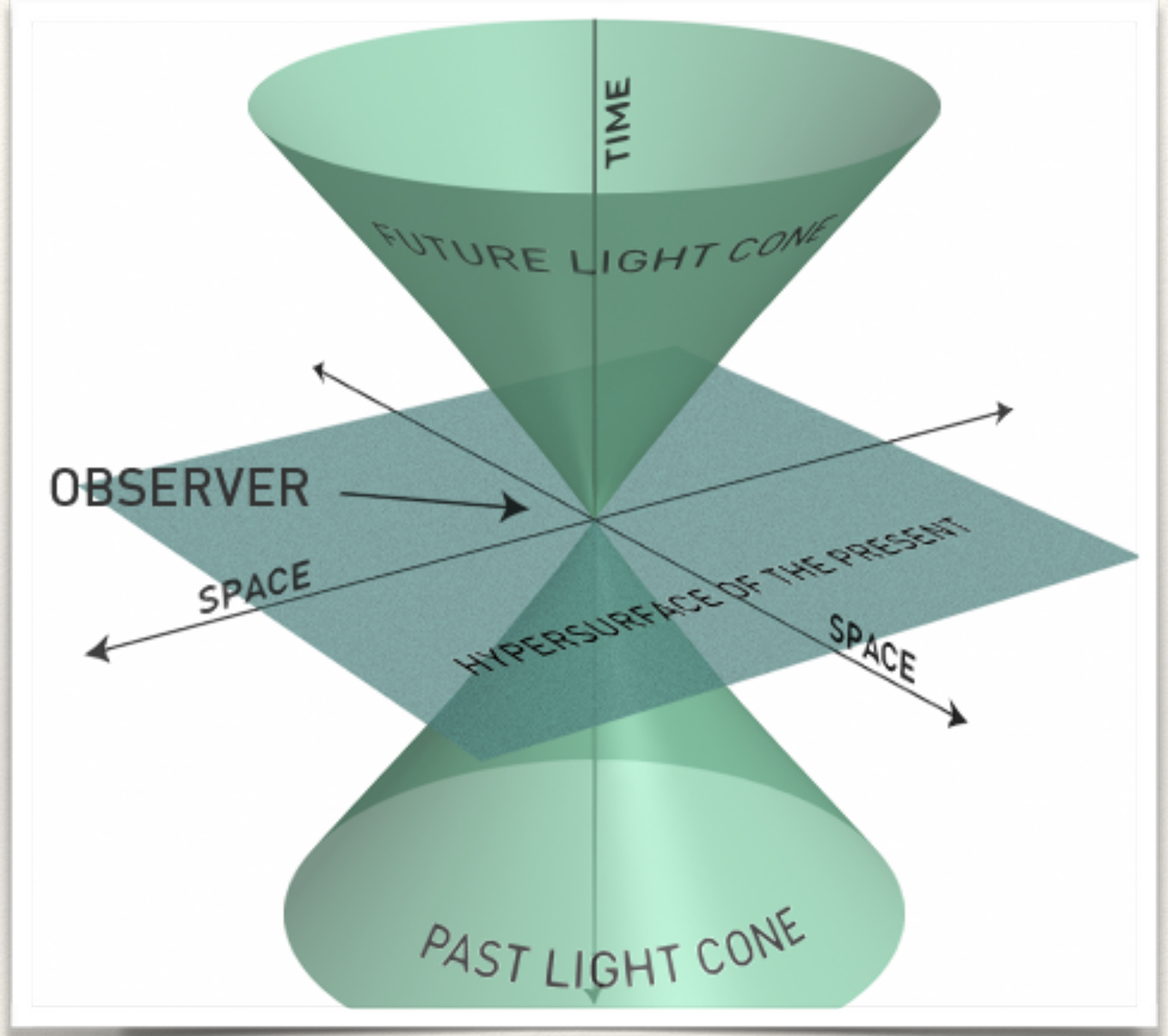
Coulomb's law in the electron's rest frame



Electromagnetic Geodesics

$$k(\vec{x}) \approx \frac{3}{q}|\underbrace{\vec{E}_{\text{ext}}(\vec{x}) \times (\vec{x} - \vec{x}_0(t))}|$$

Never singular, not even for point-like particles



$$k(\vec{x}_0(t + \Delta t)) \approx \frac{3(\Delta t)^2}{2q}|\vec{E}_{\text{ext}} \times \vec{a}_0(t)|$$

$= 0 \iff$ The charge accelerates along an **"electromagnetic geodesic"** (lines of zero curvature)

Charges accelerate along **trajectories of least electromagnetic curvature**

Thank you!

read more at

www.yaronhadad.com