The Hidden Geometry of Electromagnetism

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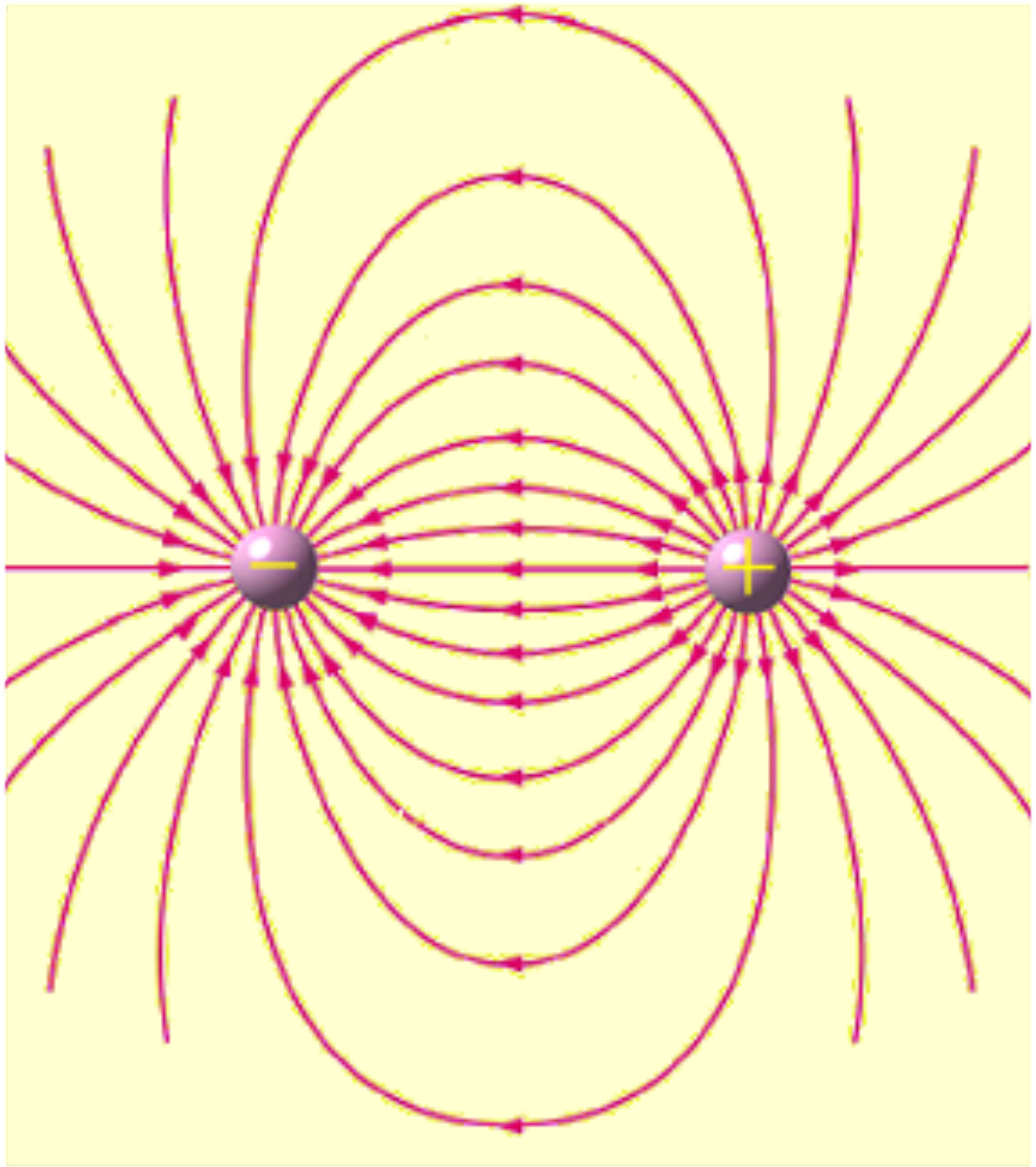
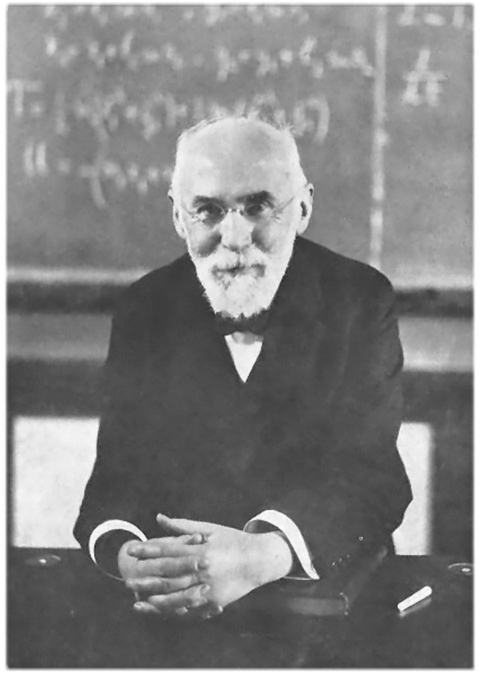
Field Lines

Michael Faraday



The Problem of Radiation-Reaction

The Lorentz Force (LF) Eq: $m\dot{u}^\alpha = -eF^{\alpha\beta}_{\text{ext}}u_\beta$



Lorentz 1892

The rate at which energy is radiated away from the electron is

$$\mathcal{R} = -m\tau_0\dot{u}^\alpha\dot{u}_\alpha$$

$$\tau_0 = \frac{2}{3}\frac{e^2}{m} = 6.24 \times 10^{-24}\,\text{s}$$

$\Longrightarrow$ an accelerating charge loses energy.

This effect is not included in the Lorentz Force equation. The rate at which energy-momentum is emitted by radiation:

$$\frac{dP^\alpha}{d\tau} = \mathcal{R}u^\alpha$$

A Plentitude of Models…

Lorentz-Abraham-Dirac (1938)	$m\dot{u}^\alpha = -eF^{\alpha\beta}u_\beta + m\tau_0\left[\ddot{u}^\alpha + \dot{u}^2 u^\alpha\right]$
Landau-Lifshitz (1952)	$m\dot{u}^\alpha = -eF^{\alpha\beta}u_\beta - e\tau_0\left[F^{\alpha\beta}_{,\gamma}u_\beta u^\gamma - \frac{e}{m}\left(F^{\alpha\beta}F_{\beta\gamma}u^\gamma - F^{\beta\gamma}F_{\gamma\delta}u^\delta u_\beta u^\alpha\right)\right]$
E…	… $\dot{u}_\gamma u^\alpha\big]$
Mo…	… $u_\gamma u^\alpha\big]$
C…	… $)u_\beta(\tau)$
Caldirola-Yaghjian (1992)	$m\dot{u}^\alpha = -eF^{\alpha\beta}(\tau)u_\beta(\tau) - \frac{m}{\tau_0}\left[u^\alpha(\tau-\tau_0) - u^\alpha(\tau)u_\beta(\tau)u^\beta(\tau-\tau_0)\right]$

Effects of Radiation-Reaction in Relativistic Laser Acceleration

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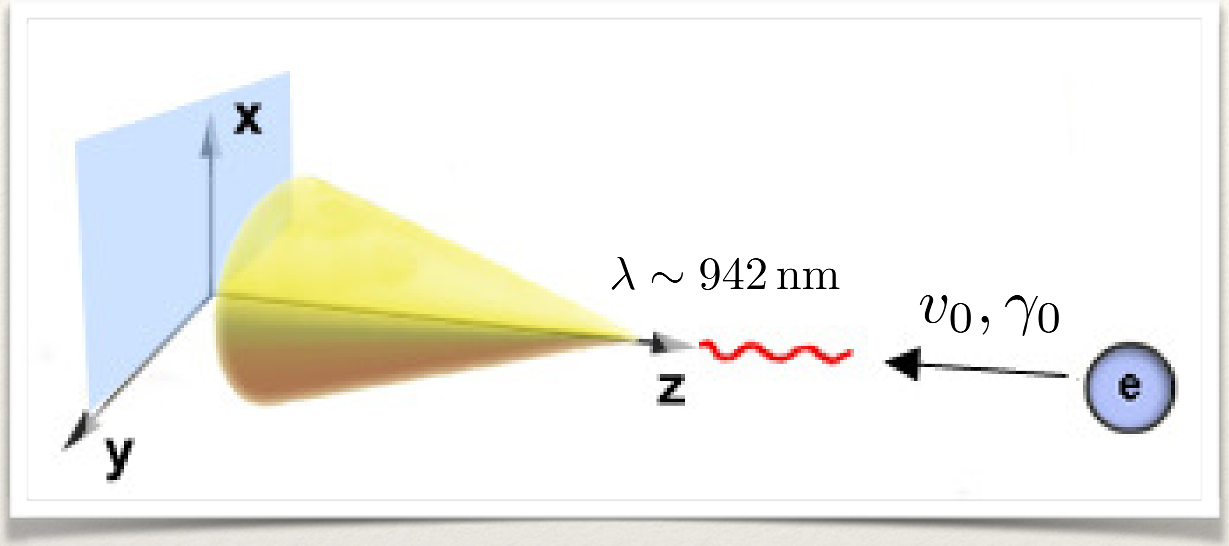
(Dated: 14 November, 2010)

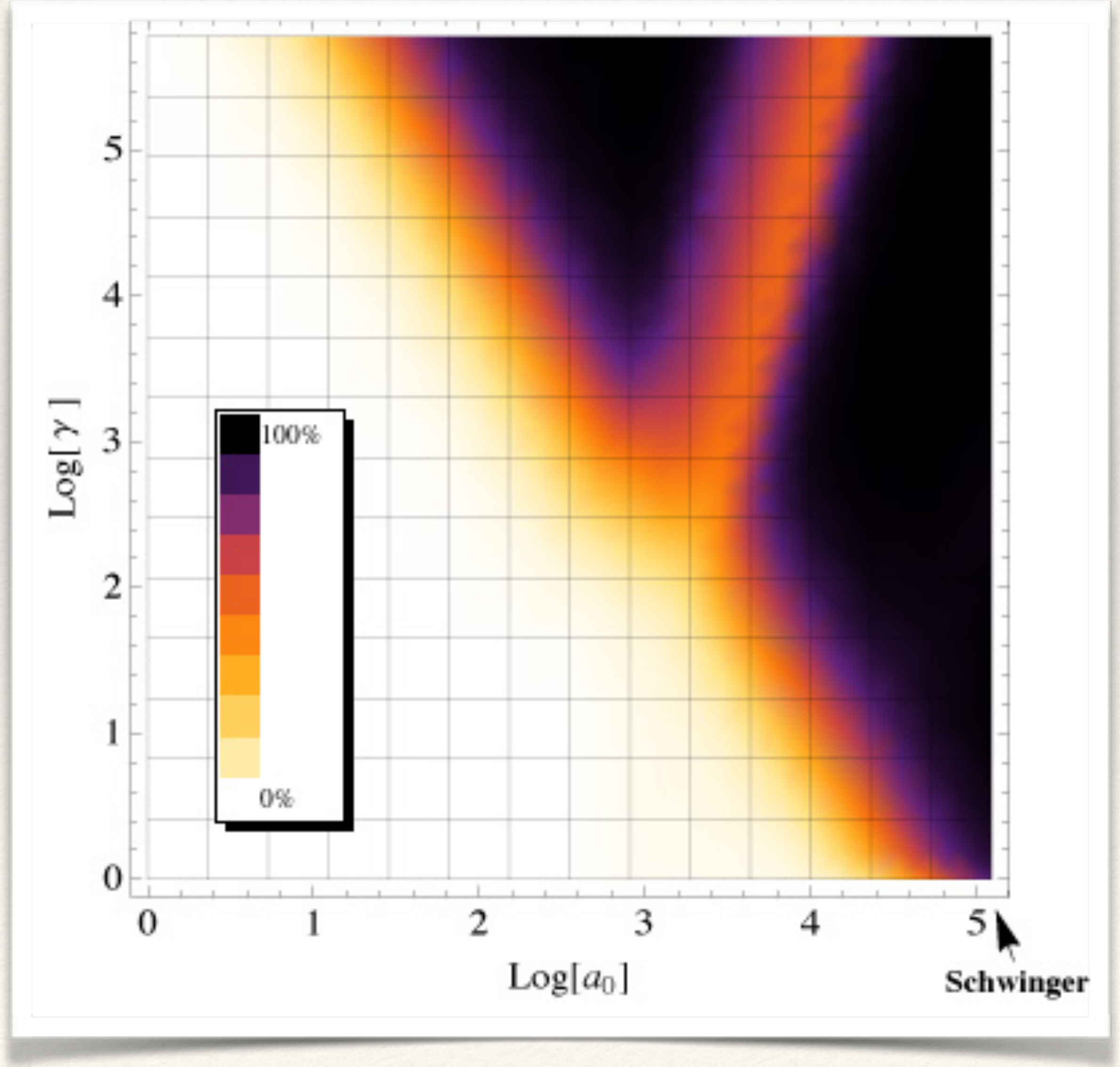
At least 7 more models were proposed….

Radiation-Reaction in Modern Experiments



Radiation-reaction criterion:

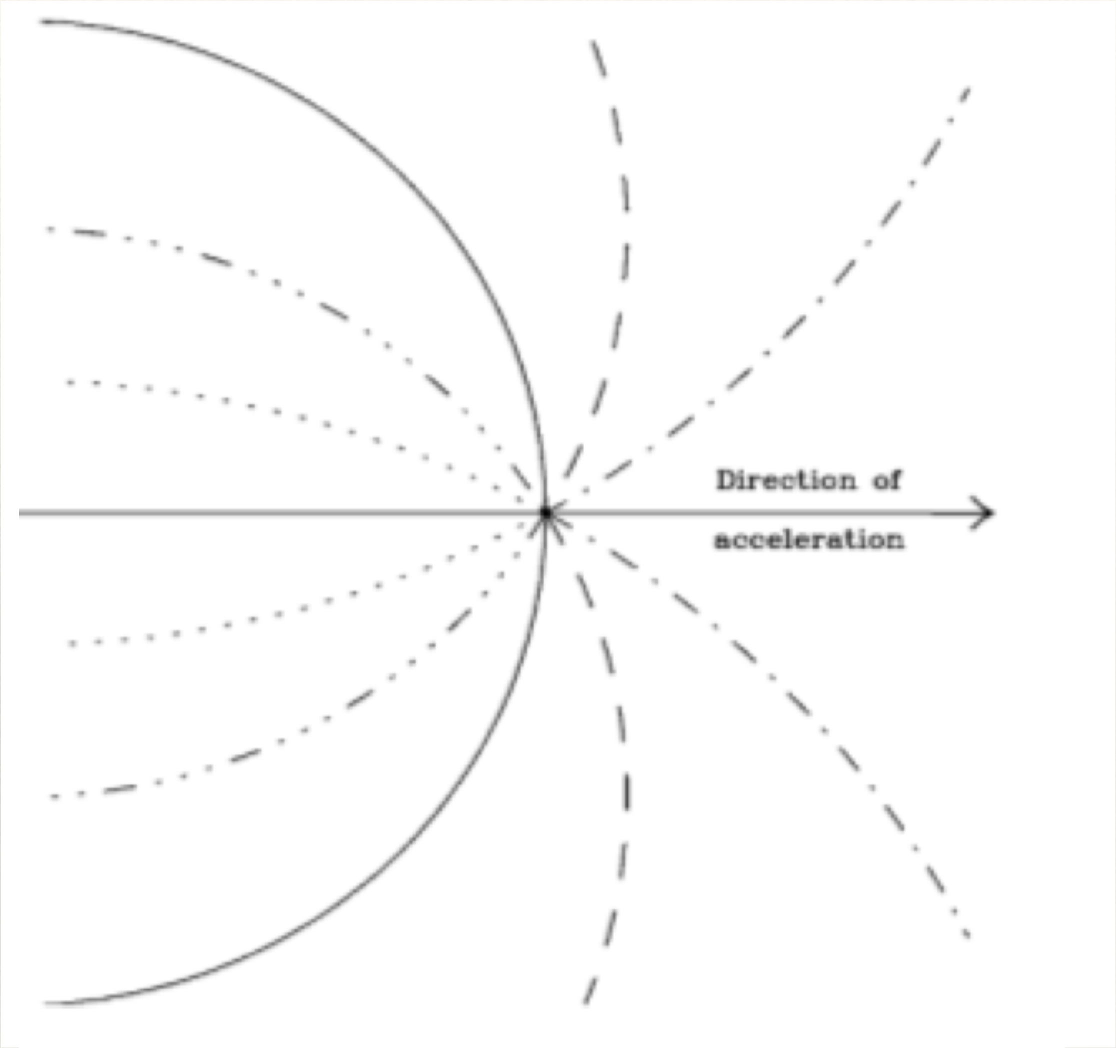
$$\gamma_0 a_0^2 \sim 10^8$$



The dimensionless intensity of a laser beam: $a_0 = \dfrac{eA_0}{m}$

A_0 = wave amplitude

Field Lines Curvature



Charge acceleration $\Longleftarrow$ $\Longrightarrow$ Field lines curvature

Electric Curvature

The curvature of a curve $\vec{\gamma}$

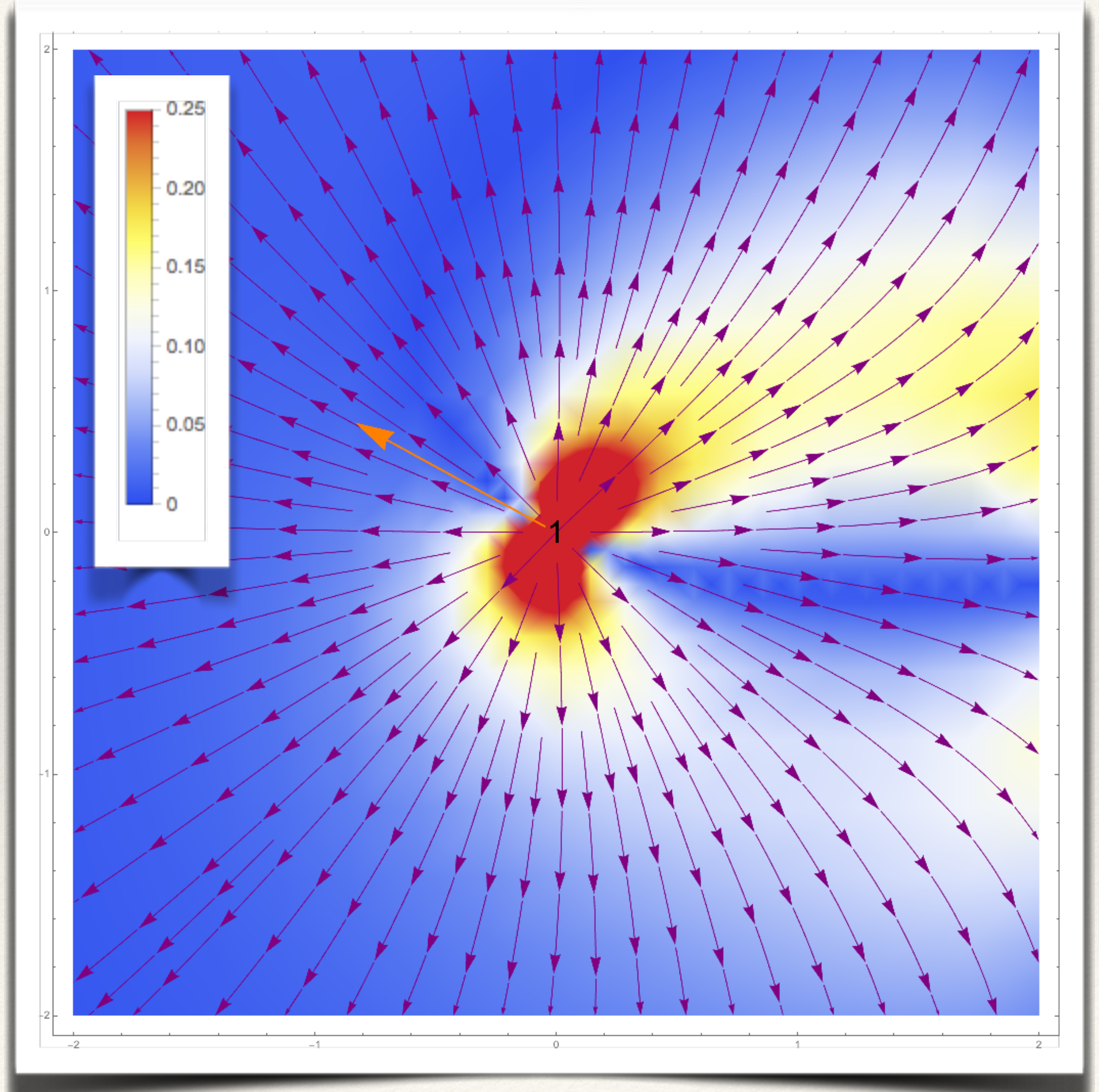
$$k = \frac{|\vec{\gamma}' \times \vec{\gamma}''|}{|\vec{\gamma}'|}$$

For an electric field line:

$$k = \frac{|\vec{E} \times (\vec{E} \cdot \nabla)\vec{E}|}{|\vec{E}|^3}$$

$$\vec{E}(\vec{x}) = \underbrace{\vec{E}_{\text{self}}(\vec{x})} + \vec{E}_{\text{ext}}(\vec{x})$$

Coulomb's law in the electron's rest frame



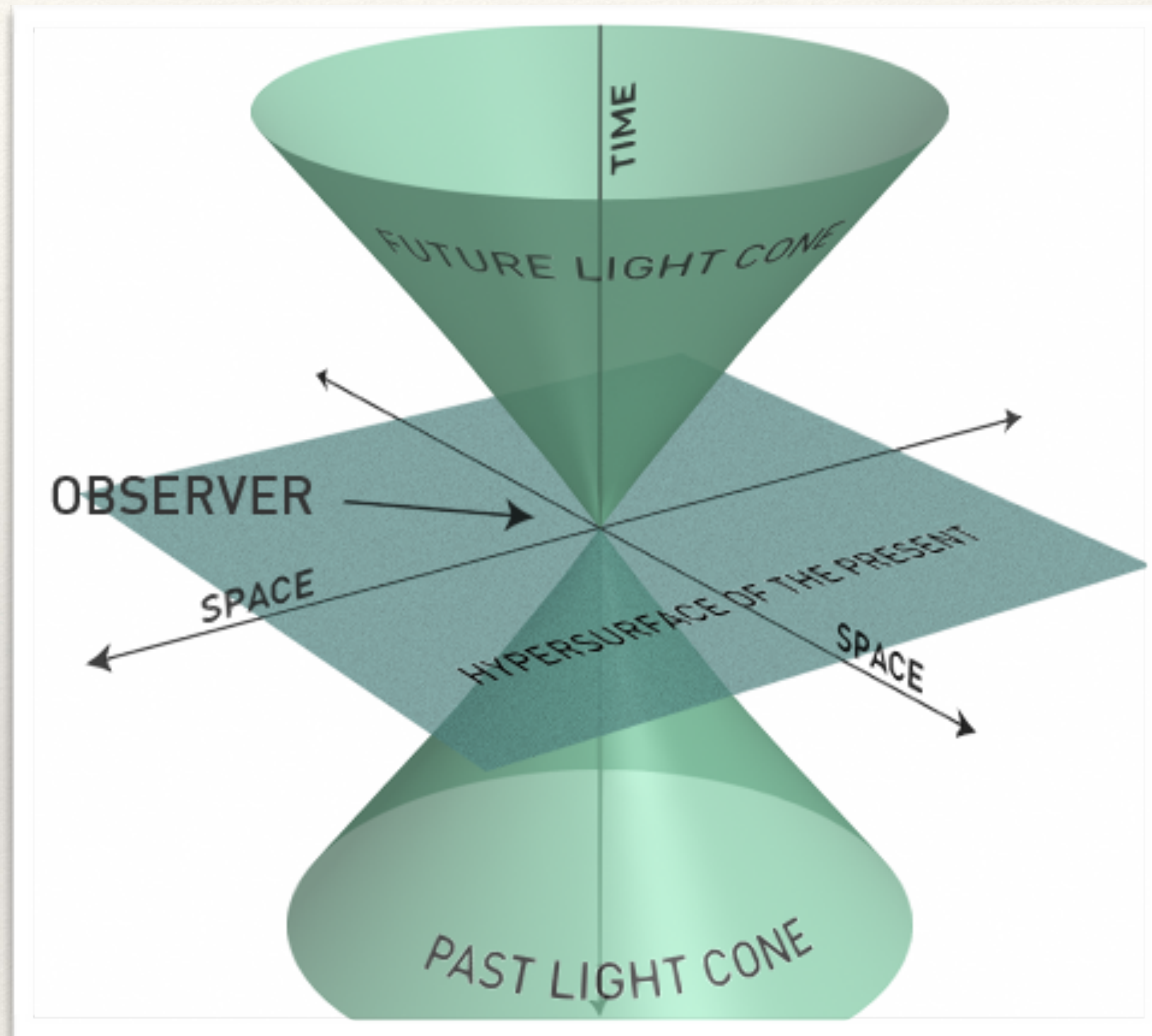
Electromagnetic Geodesics

$$k(\vec{x}) \approx \frac{3}{q} |\underbrace{\vec{E}_{\rm ext}(\vec{x}) \times (\vec{x} - \vec{x}_0(t))}|$$

Never singular, not even for point-like particles

$$k(\vec{x}_0(t+\Delta t)) \approx \frac{3(\Delta t)^2}{2q} |\vec{E}_{\rm ext} \times \vec{a}_0(t)|$$

$= 0 \iff$ The charge travels along an **"electromagnetic geodesic"** (lines of zero curvature)



Charges travel along **trajectories of least electromagnetic curvature**

Grazie mille!

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