December 7, 2009

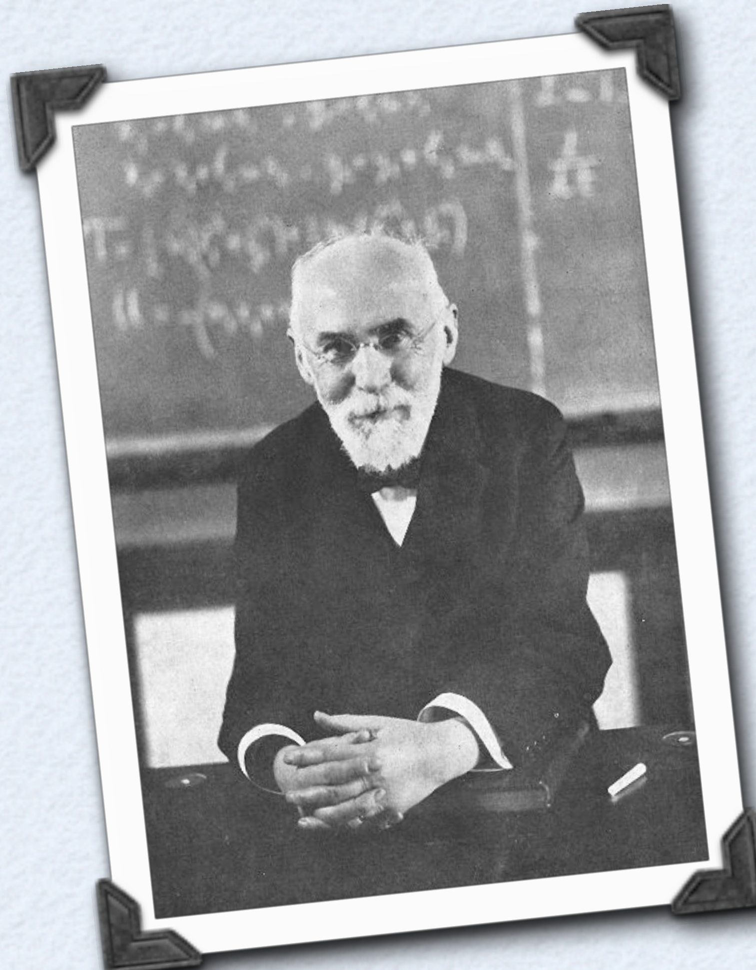
Radiation and Acceleration

Probing the radiation-reaction dominated regime

Yaron Hadad

High Acceleration Collaboration

- D. Habs
- Y. Hadad
- B. M. Hegelich
- L. Labun
- J. Rafelski
- H. Ruhl
- P. Thirolf



Arising from a work in progress on solutions of Landau-Lifshitz equation

Outline

- The problem of radiation-reaction
- Lines of attack
- The radiation-reaction dominated regime criterion
- The solution of the Landau-Lifshitz equation
- Radiation emission & spectrum

The Problem of Radiation-Reaction

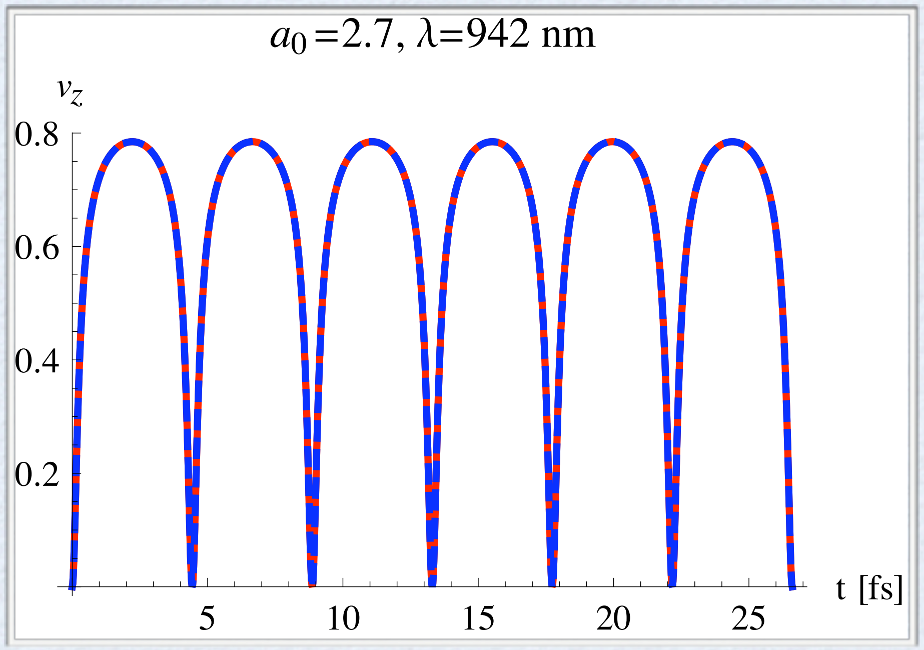
The Lorentz force equation: $m\dot{u}^\alpha = -eF^{\alpha\beta}u_\beta$

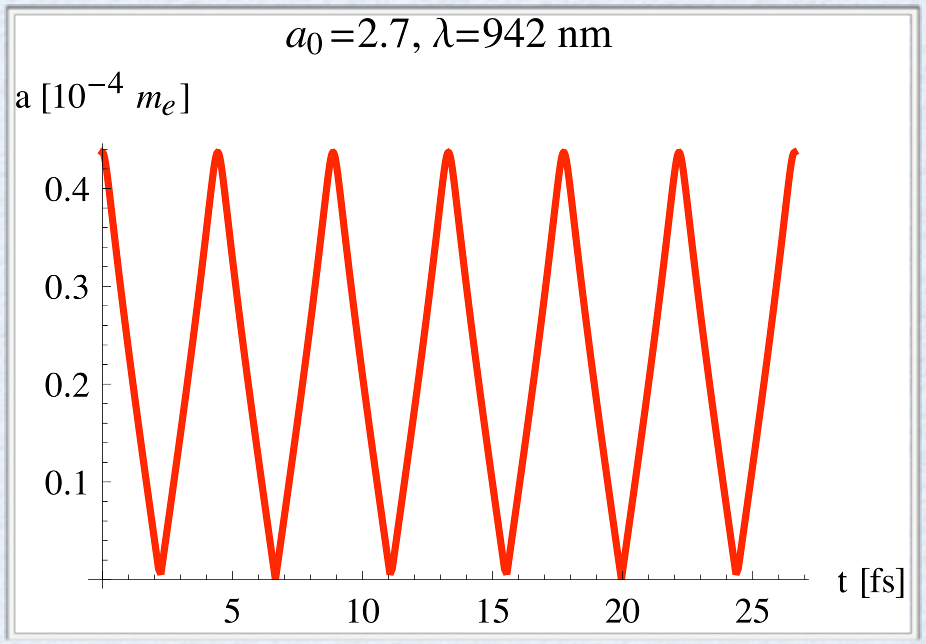
The rate at which energy is radiated away from the electron is: $\mathcal{R} = -m\tau_0\dot{u}^\alpha\dot{u}_\alpha$ where $\tau_0 = \frac{2}{3}\frac{e^2}{m} = 6.24\times10^{-24}$ s

Therefore an accelerating charge loses energy.
This effect is not included in the Lorentz force equation!

Question

Is radiation-reaction significant as a force?





Electron emits much less than 1% of its rest energy

Current Radiation-Reaction Models

Dirac: Maxwell equations and energy conservation give:

$$m\dot{u}^{\alpha} = \underbrace{-eF^{\alpha\beta}u_{\beta}}_{F_{Lorentz}} + \underbrace{m\tau_0\left[\ddot{u}^{\alpha} + \dot{u}^2 u^{\alpha}\right]}_{F_{RR}}$$

this is called the LAD Equation (Lorentz-Abraham-Dirac).

The 3rd order time derivative requires another initial condition (initial acceleration) and results in infinitely many non-physical solutions...

Dirac replaced the additional condition with an "asymptotic condition". Instead of giving the initial acceleration, give the final acceleration.

Current Radiation-Reaction Models

Landau-Lifshitz: Usually $F_{RR} \ll F_{Lorentz}$

So in the first order: $m\dot{u}^{\alpha} = -eF^{\alpha\beta}u_{\beta} + \cancel{m\tau_0\left[\ddot{u}^{\alpha} + \dot{u}^2 u^{\alpha}\right]}$

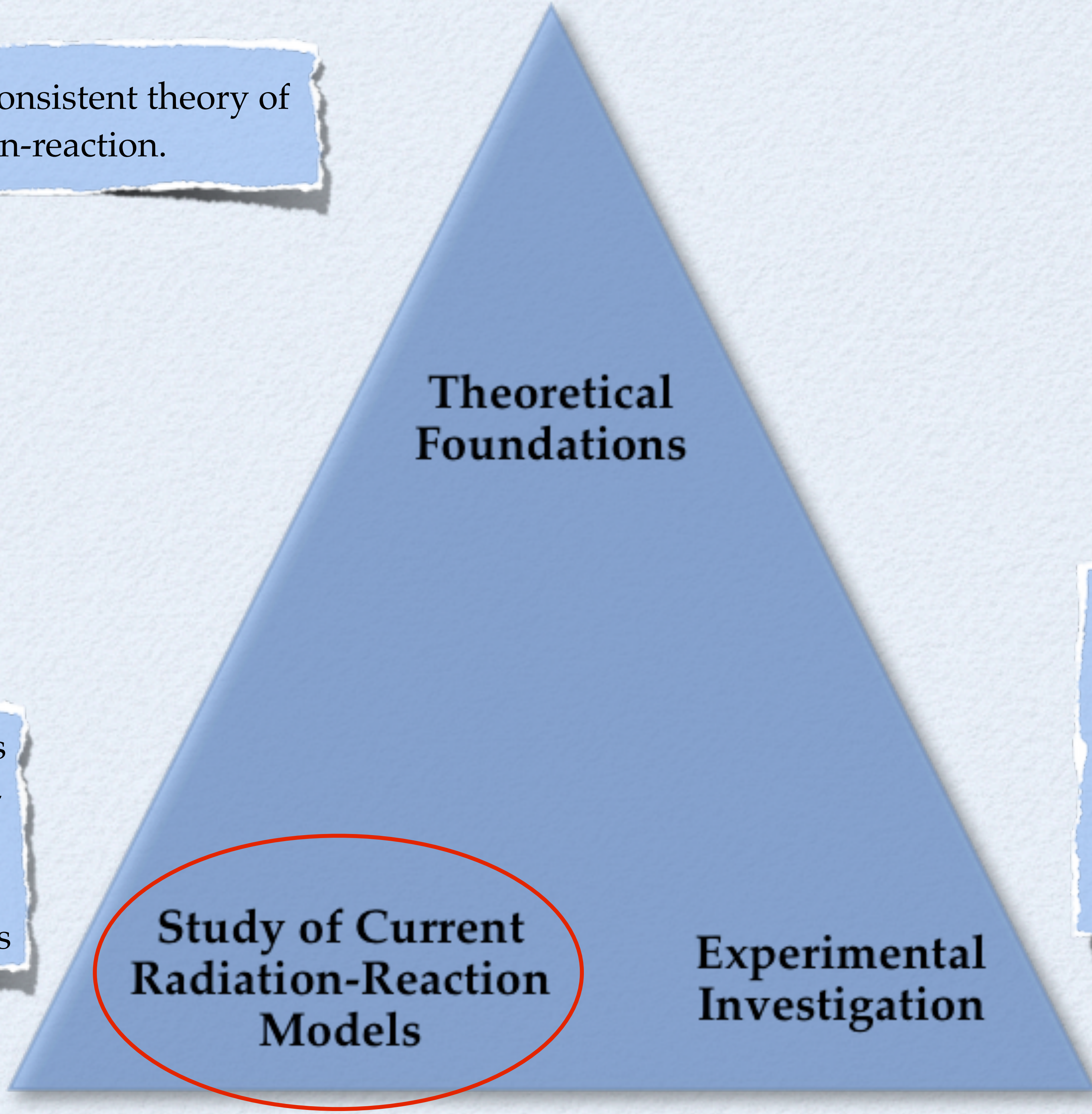
We can "get rid" of the third derivative in the radiation-reaction force, and get the Landau-Lifshitz equation:

$$m\dot{u}^{\alpha} = -eF^{\alpha\beta}u_{\beta} - e\tau_0\left\{F^{\alpha\beta}_{,\gamma}u_{\beta}u^{\gamma} - e/m\left[F^{\alpha\beta}F_{\beta\gamma}u^{\gamma} - F^{\beta\gamma}F_{\gamma\delta}u^{\delta}u_{\beta}u^{\alpha}\right]\right\}$$

There are at least five other possible models (e.g. Caldirola, Mo-Papas, Eliezer), but only Landau-Lifshitz Eq. is considered to be theoretically reasonable [Wald et al. 2009]

Lines of Attack for Solving the Problem

Obtain a fully consistent theory of radiation-reaction.



What do current models predict about radiation-reaction?

Find and study solutions

Test radiation-reaction in the lab.

Up to the present time, the radiation-reaction force remains experimentally untested

The Radiation-Reaction Dominated Regime (RRDR)

The rate at which energy is radiated for the Landau-Lifshitz equation:

$$\mathcal{R} = -\frac{2}{3}e^2\frac{(k\cdot u)^4}{\omega^2}\Big\{a_0^2\hat{A}'^2 \qquad \textcolor{red}{\text{(Lorentz)}}$$

$$\underbrace{+\,(\omega\tau_0)\left[a_0^2\hat{A}''\cdot\hat{A}' - a_0^4\psi\hat{A}'^2\right] + O(\tau_0^2)\Big\}}$$

Landau-Lifshitz correction $\sim \omega\tau_0 a_0^4$

Radiation-reaction is important when $\omega\tau_0 a_0^2 \sim 1$

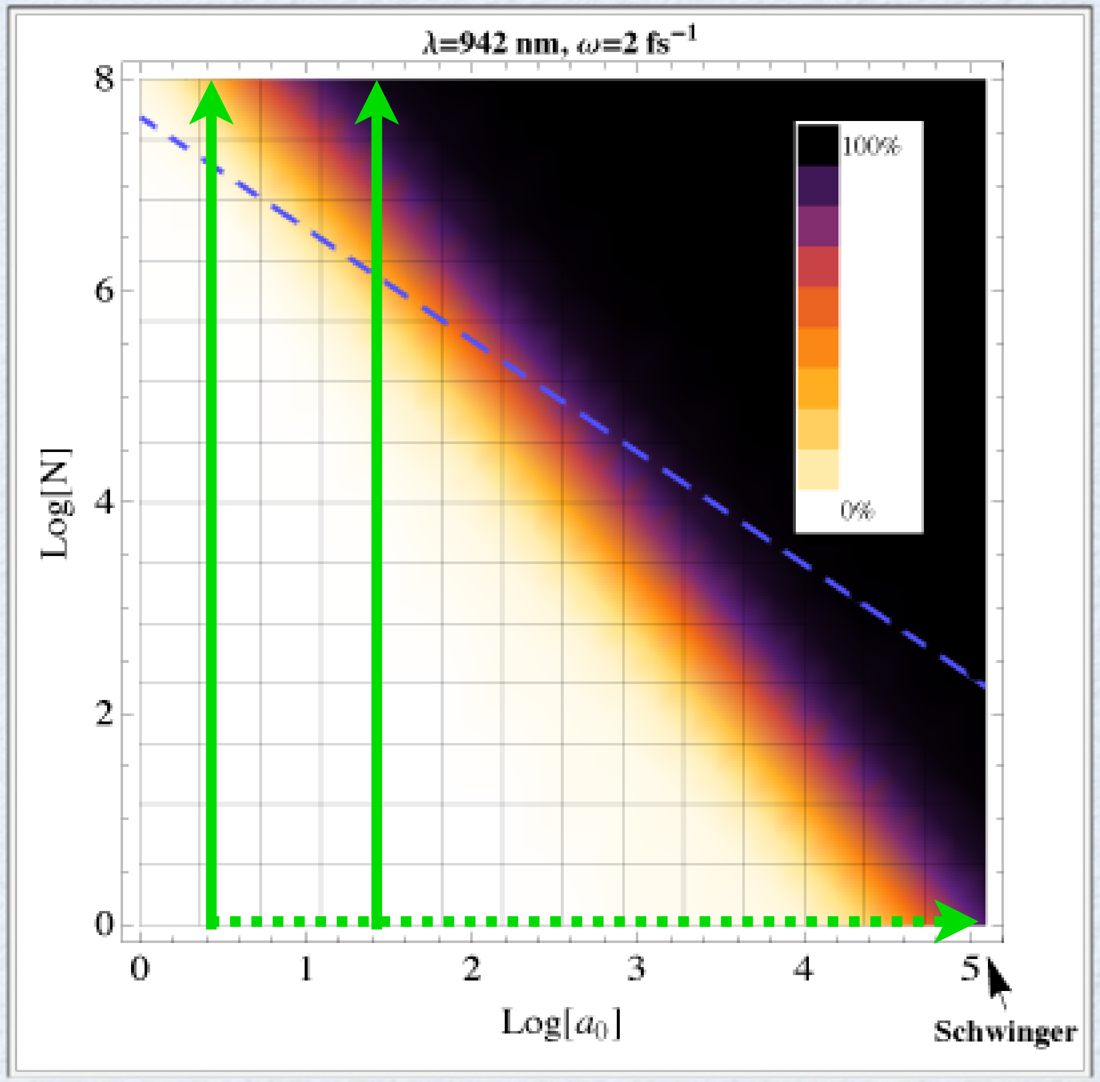
or for a single electron: $a_0 \sim 10^4$

$$\tau_0 = \frac{2}{3}\frac{e^2}{mc^3} \xrightarrow[m\to Nm]{e\to Ne} \frac{2}{3}\frac{e^2}{mc^3}N$$

for a coherent cluster of N electrons

RRDR Criterion

$$Na_0^2 \sim 10^8$$



e.g. $a_0 = 2.7$ $a_0 = 27$

The Setup

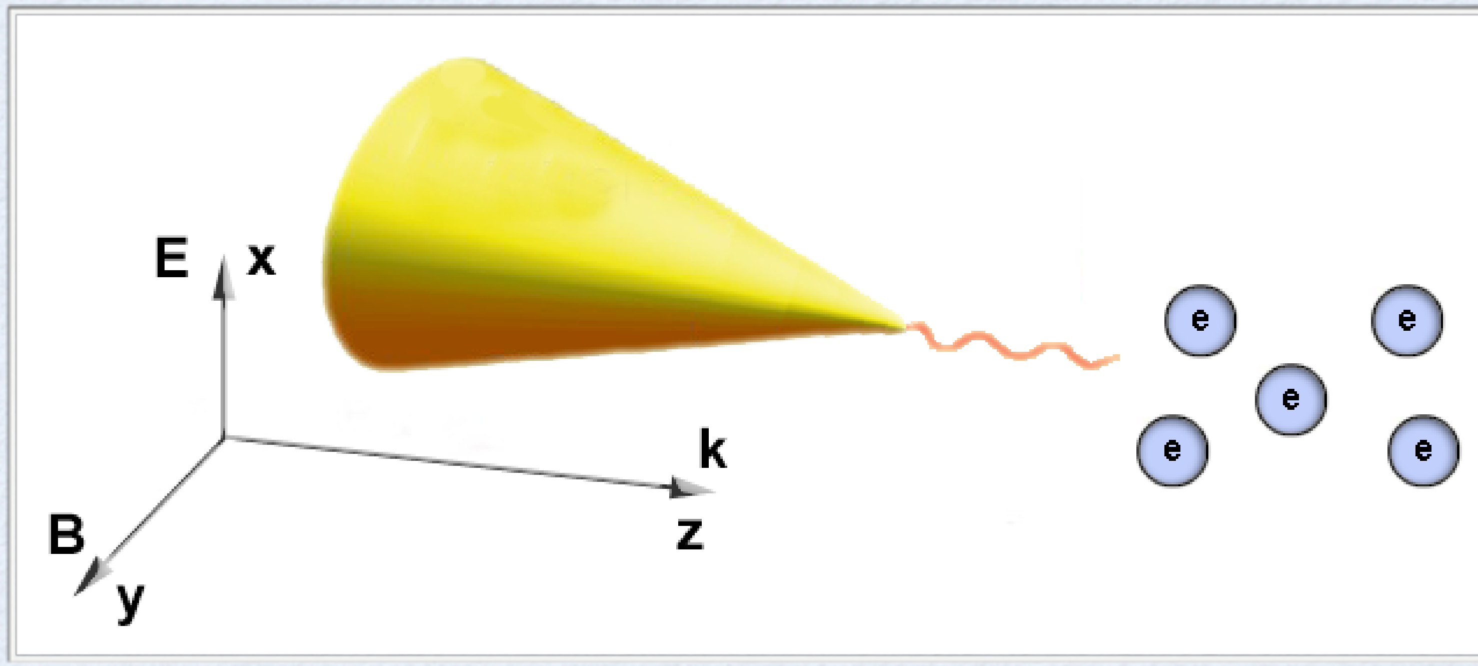
We use a linearly polarized (LP) plane wave with fields:

$$\begin{aligned}\vec{A} &= -A_0 \sin(kz - \omega t)\hat{x} \\ \vec{E} &= -\omega A_0 \cos(kz - \omega t)\hat{x} \\ \vec{B} &= -k A_0 \cos(kz - \omega t)\hat{y}\end{aligned}$$

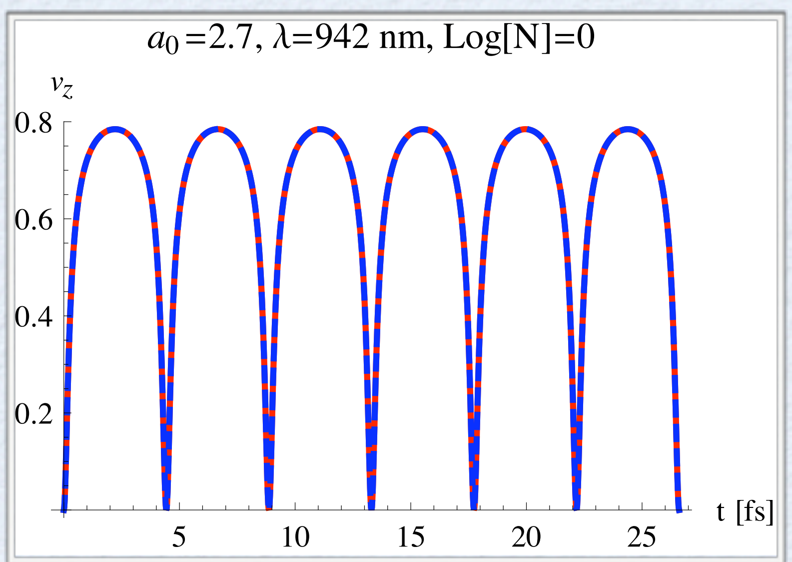
$$a_0 = 2.7 \qquad T = 26.8\,\mathrm{fs}$$

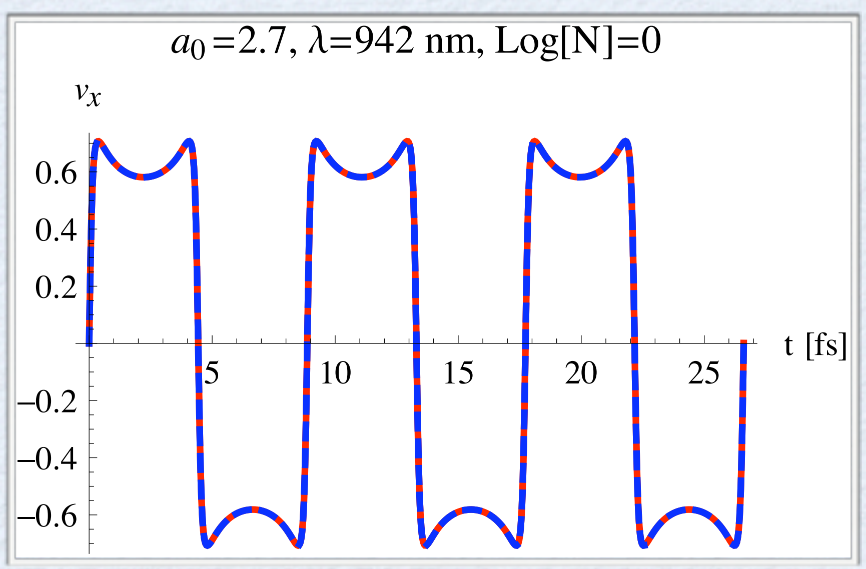
$$\lambda = 942\,\mathrm{nm} \qquad \omega = 2\,\mathrm{fs}^{-1}$$

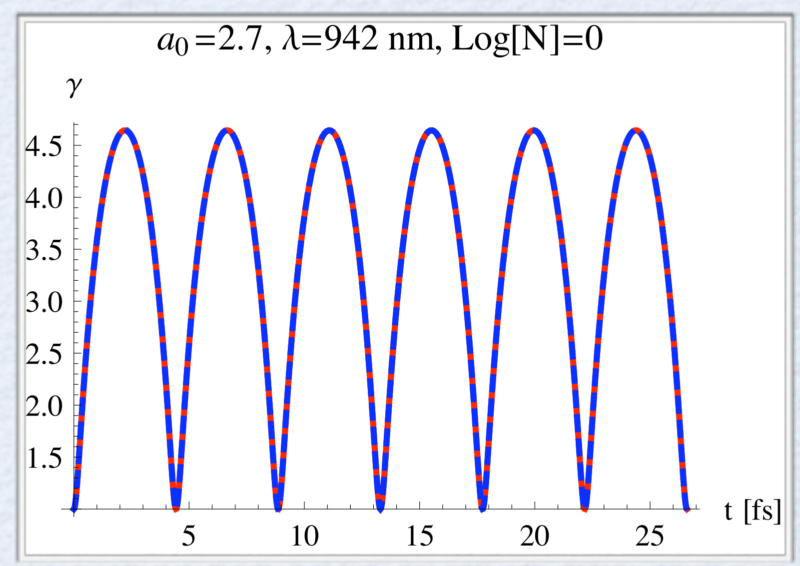
Wave Direction: $\hat{z}$ Polarization: $\hat{x}$



The usual solution of the Lorentz force equation in lab time:







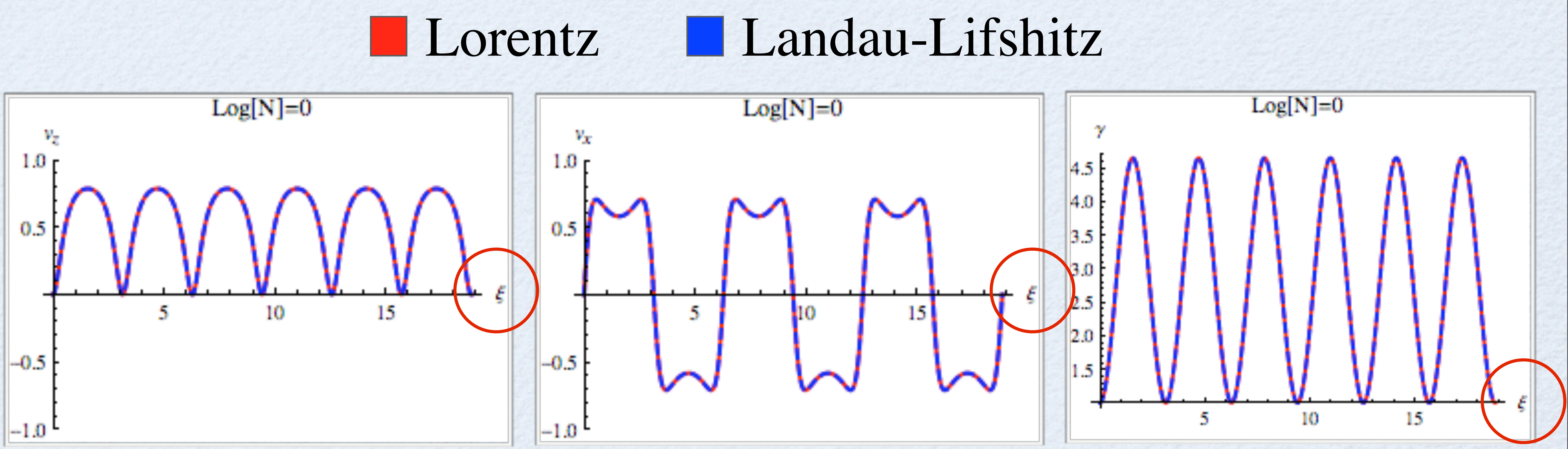
Solution of the Landau-Lifshitz Equation

Since the Landau-Lifshitz equation is nonlinear, solving in terms of the proper time is much more complicated...

Remember that the wave is a function of $\xi = k \cdot x = \omega t - \vec{k} \cdot \vec{x}$

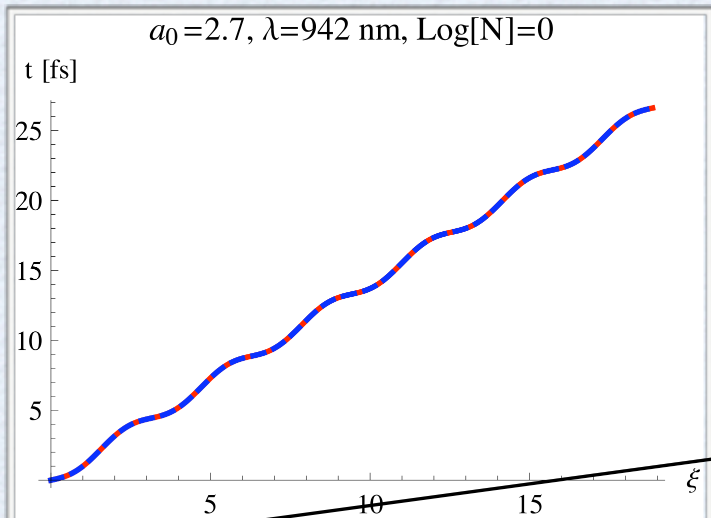
The "trick": change variables $\tau \mapsto \xi$

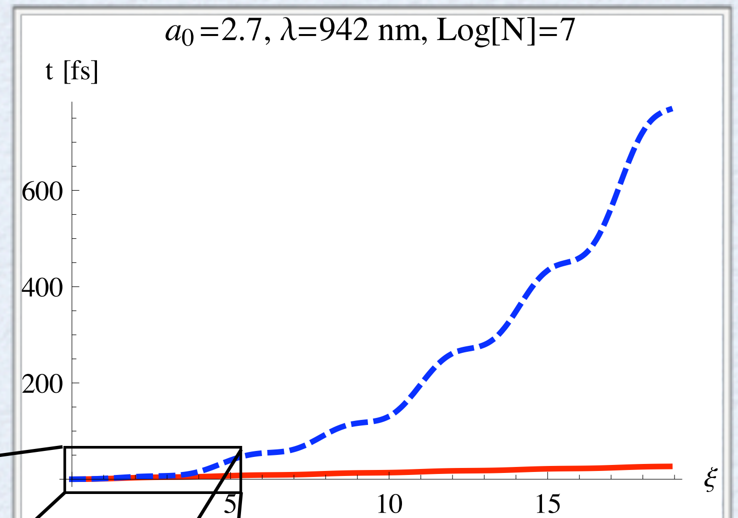
(this works much more generally than a LP plane wave)

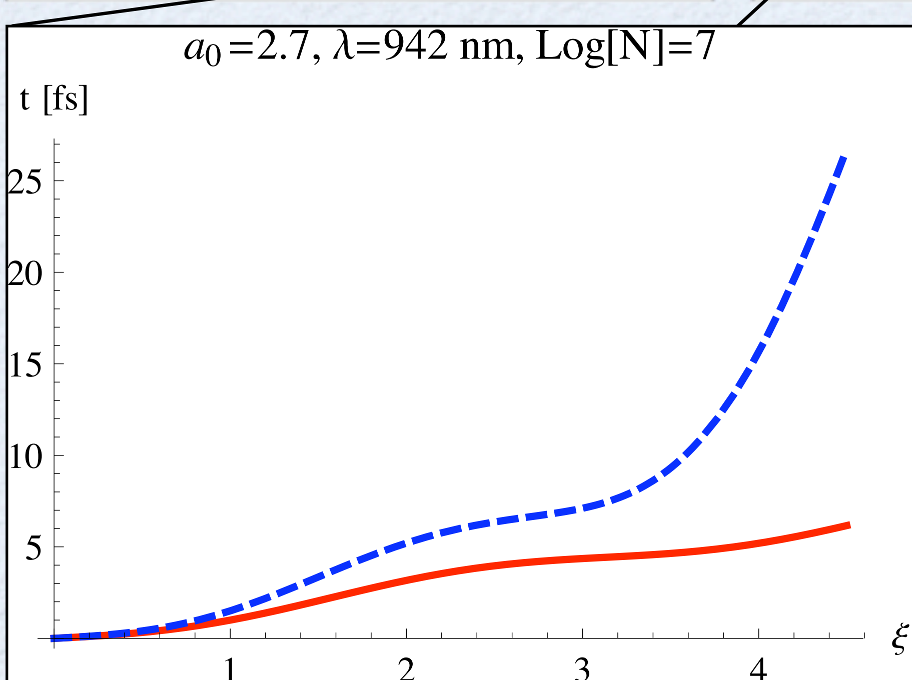


Back to Laboratory Time

There is a non-trivial relationship between lab time and the phase.

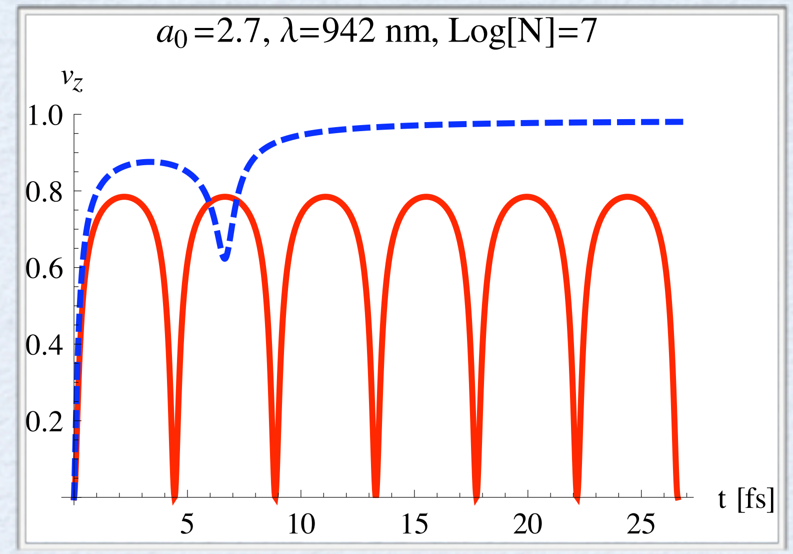




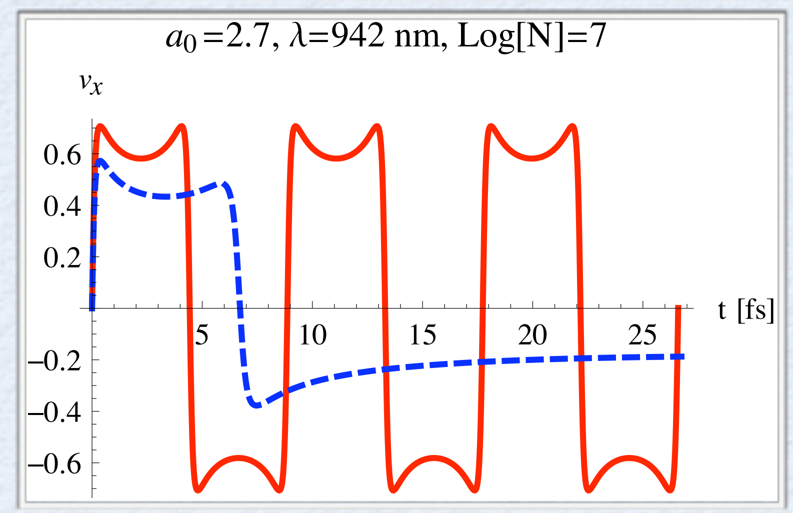


This results in a **dilation** of the periods of the electrons cluster.

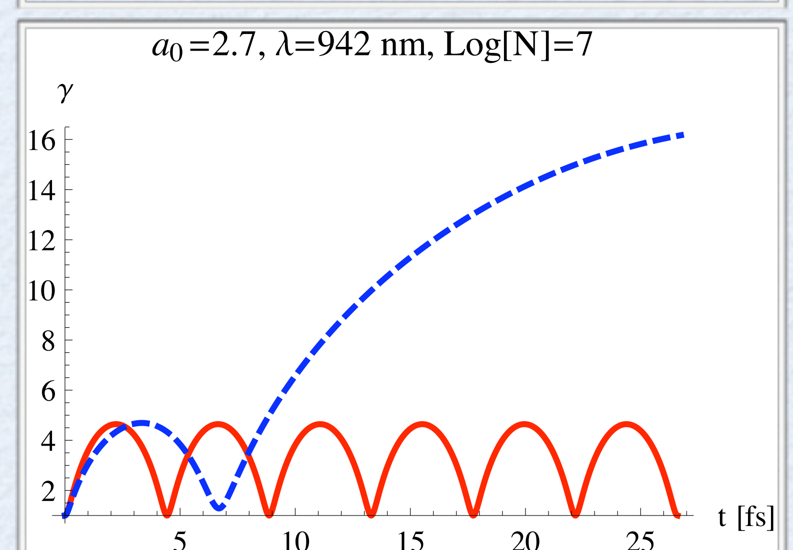
■ Lorentz ■ Landau-Lifshitz



Momentum gain in direction of wave



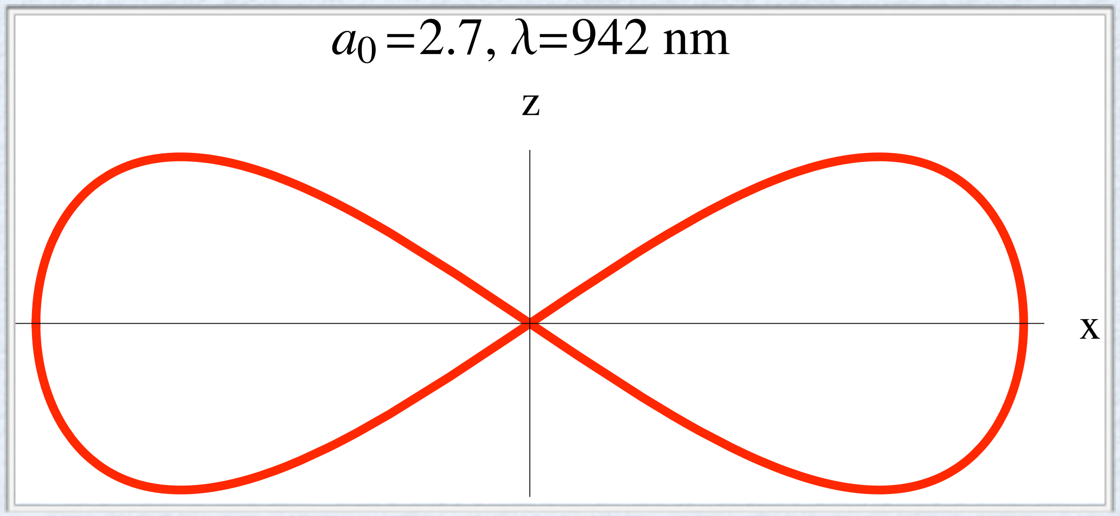
Momentum loss in direction of polarization



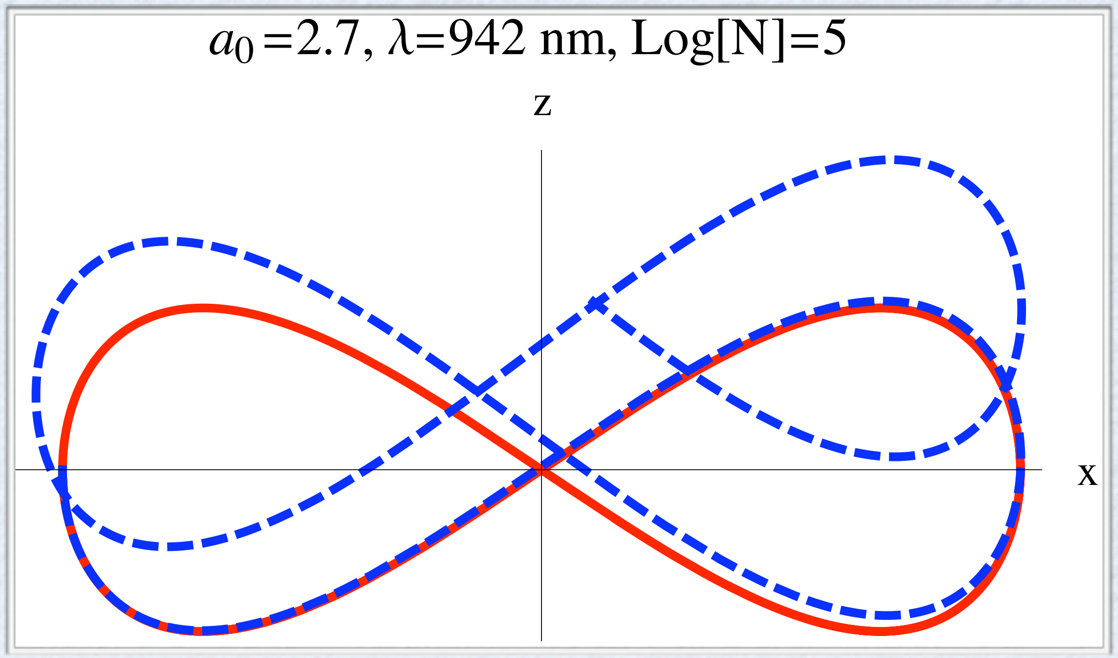
But altogether each electron in the cluster gains more energy

The Motion of the Electrons Cluster

For the Lorentz force equation, the motion in the electron's drift frame is an 'eight' figure.



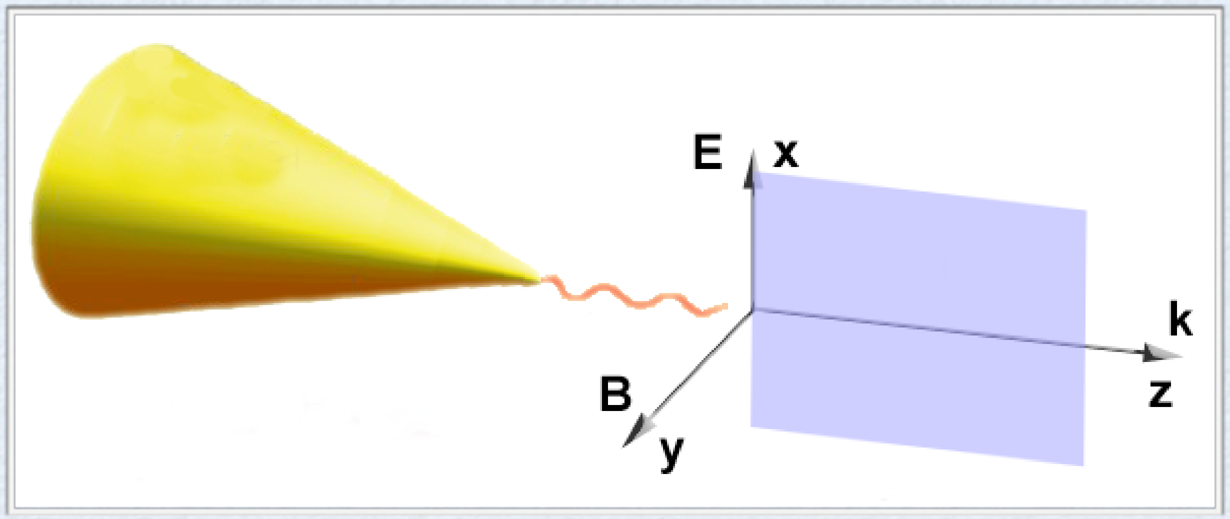
In the same frame, the electrons cluster breakout faster than a single electron:

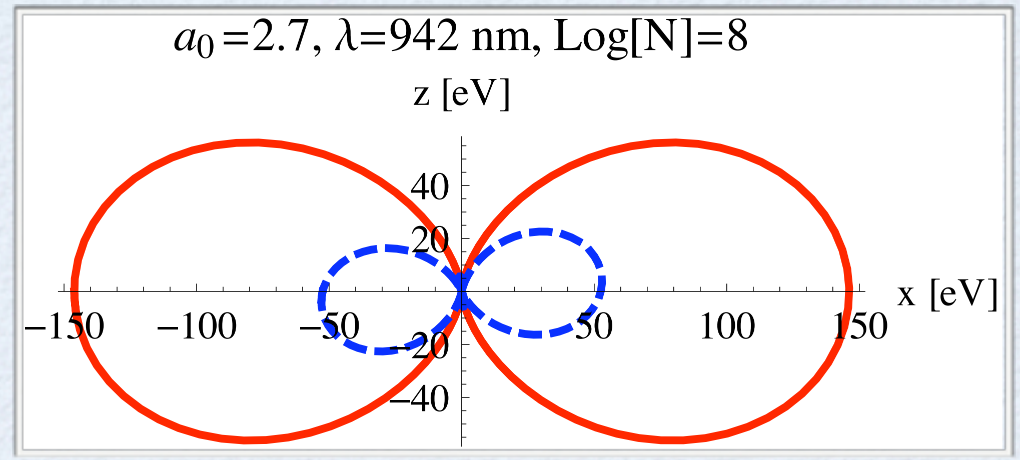


There is a small drift in the initial direction of polarization

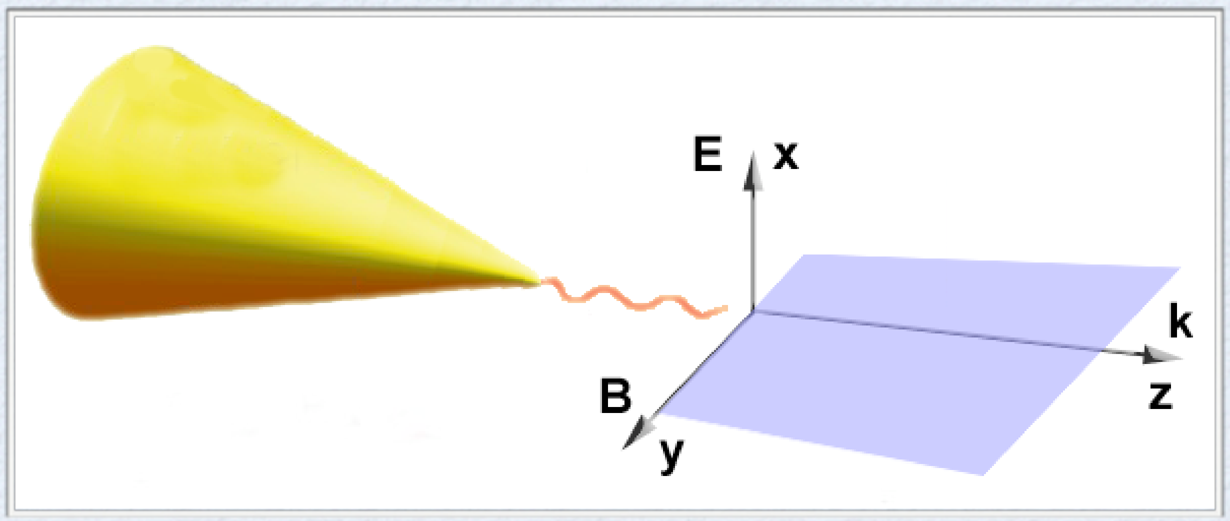
The radiation-reaction is a non-conserving force, so the motion in the (inertial) drift frame is not a close contour.

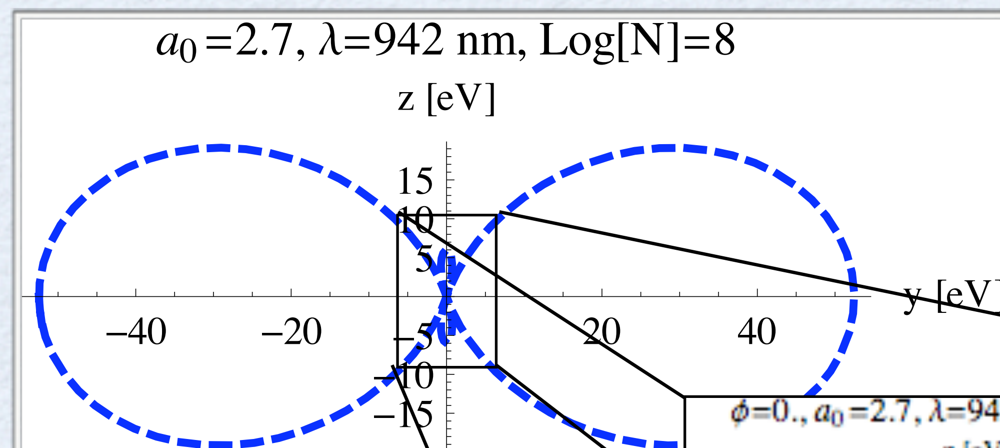
Angular Distribution of Radiation

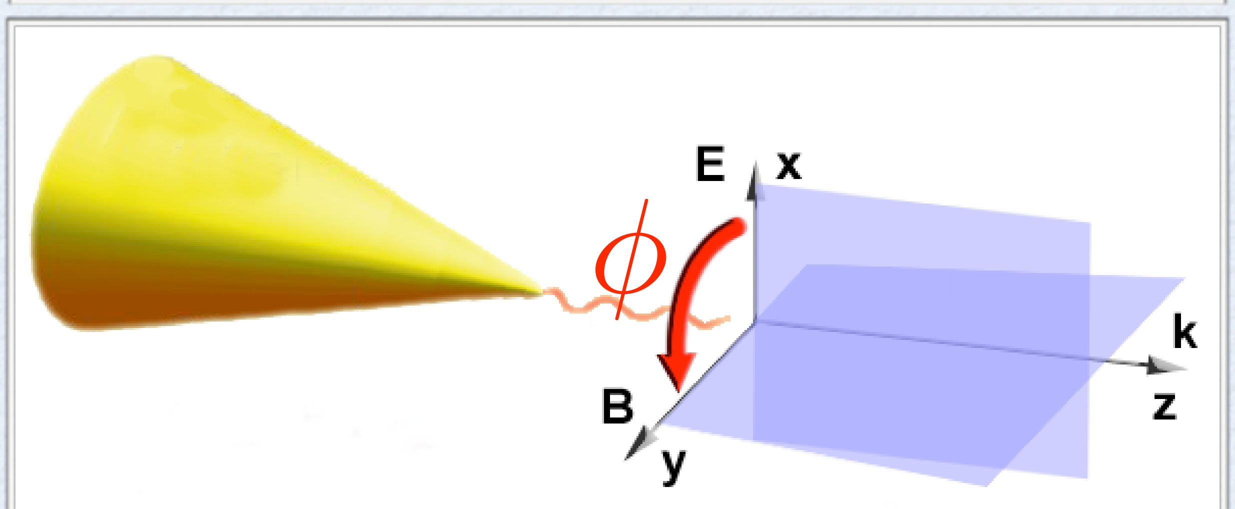




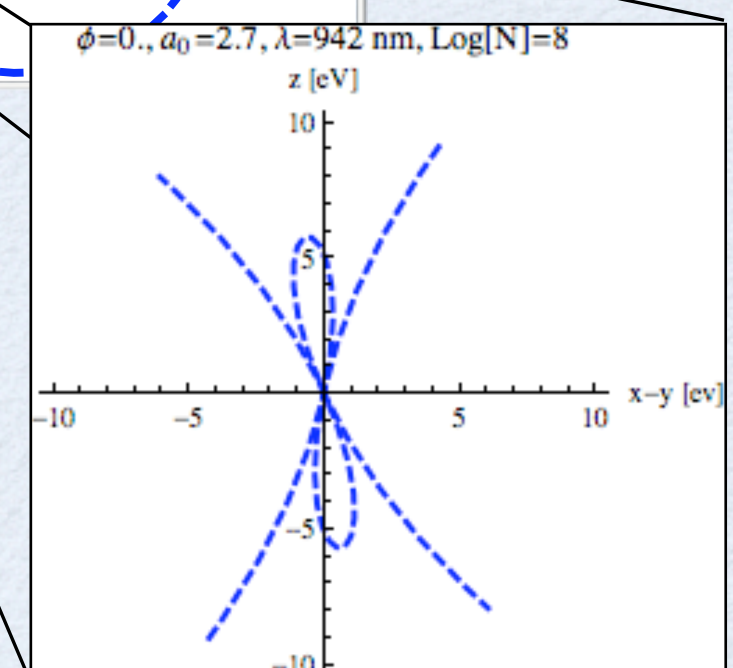
Each electron in the cluster emits less radiation



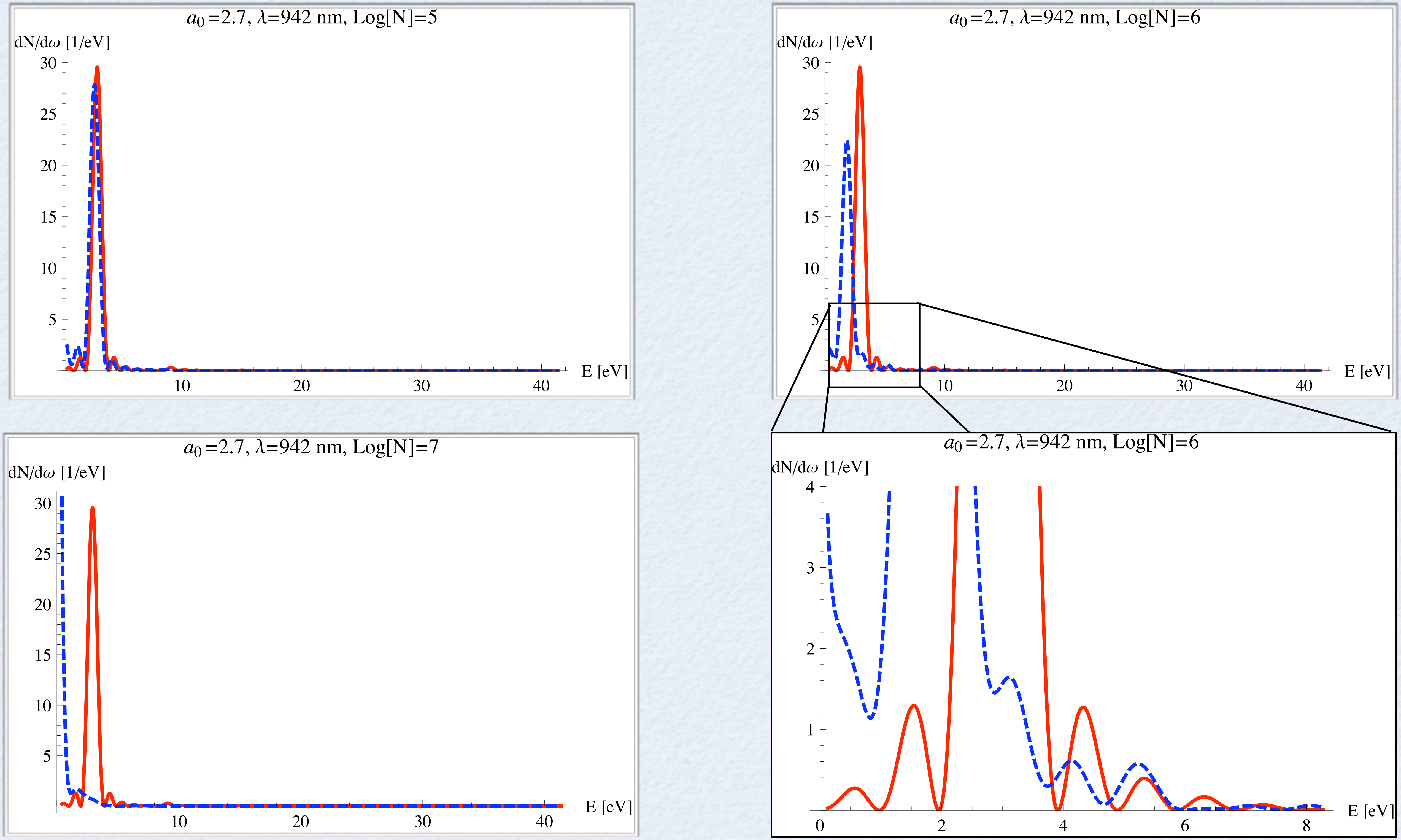




There is emission of radiation in the **longitudinal** direction

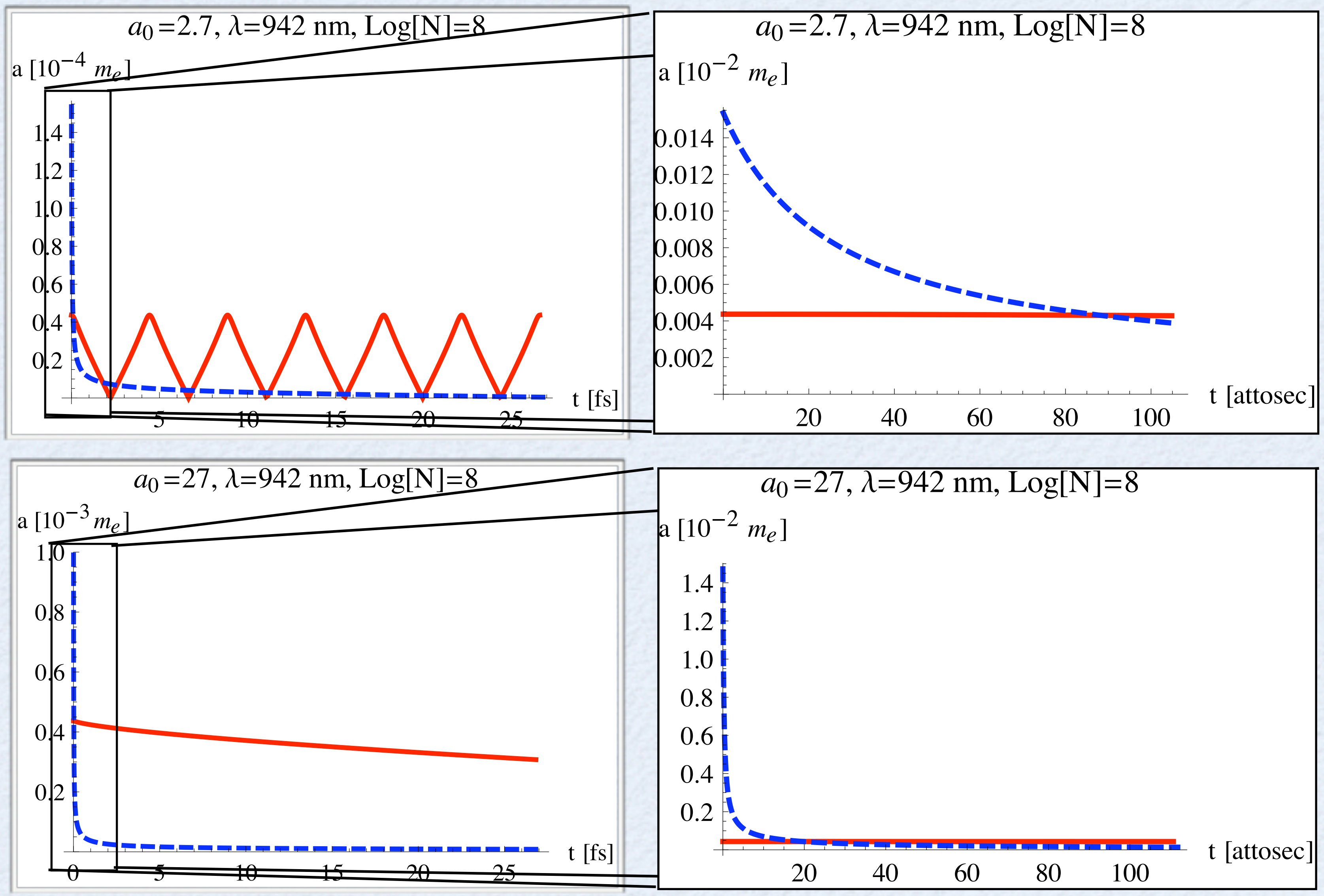


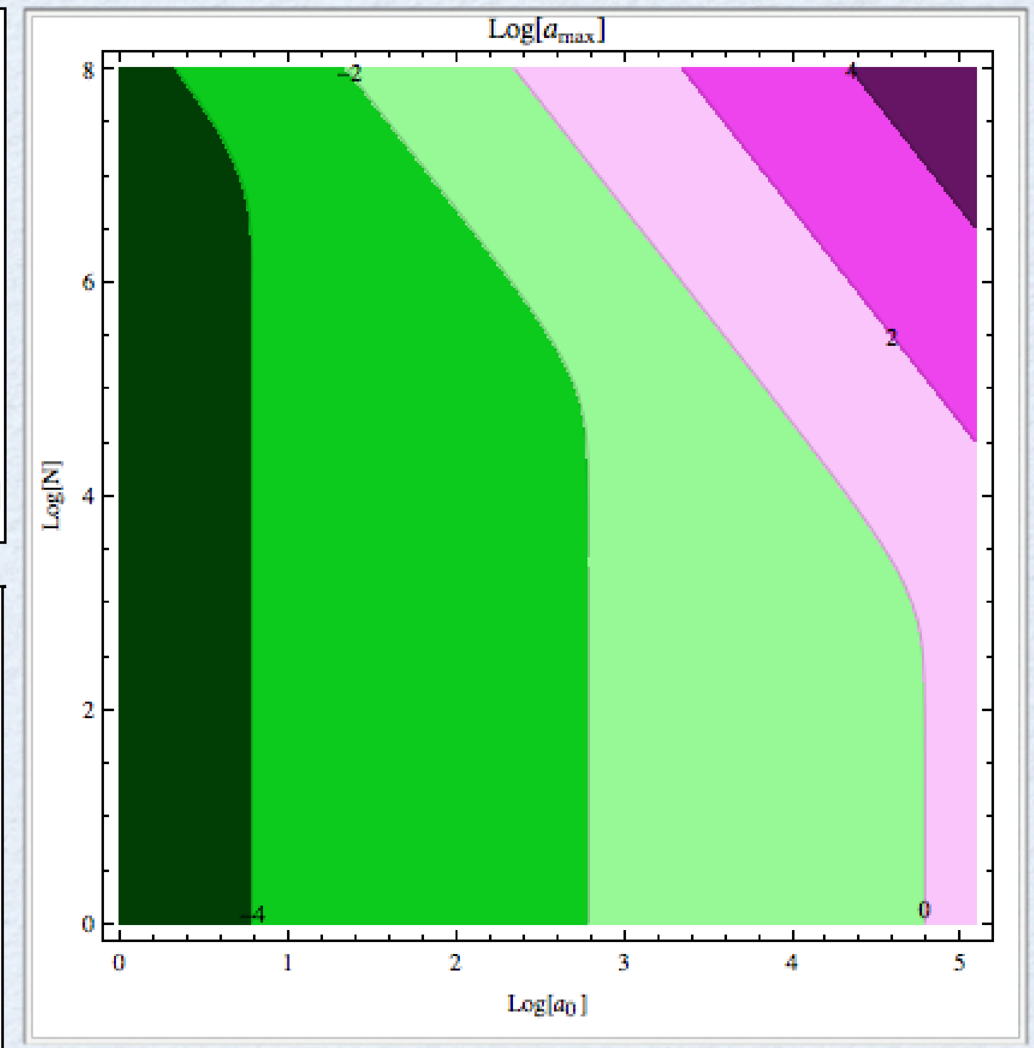
Radiation Spectrum

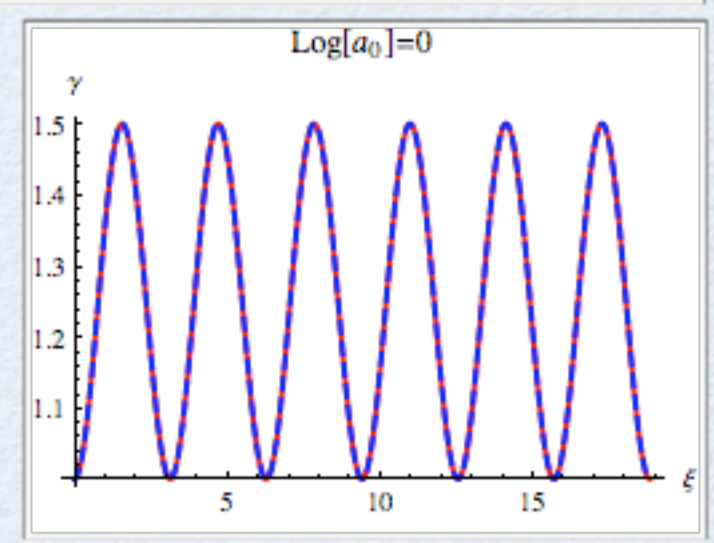


Lorentz   Landau-Lifshitz

Back to Acceleration







Can acceleration indeed be arbitrarily large?

The Landau-Lifshitz equation predicts no limit on acceleration.

References

- Dirac. Classical theory of radiating electrons. Proceedings of the Royal Society of London. Series A (1938)
- L. D. Landau and E. M. Lifshitz. The Classical theory of Fields. Pergamon, Oxford (1962)
- Gralla, Harte and Wald. Rigorous derivation of electromagnetic self-force. Physical Review D (2009) vol. 80 (2) pp. 24031
- J. Rafelski, L. Labun and Y. Hadad. Horizons of Strong Fields Physics. arXiv:0911.5556v1 [physics.acc-ph]

Thank you!

Supplemental Formulas

The relation between variables: $$\frac{d\xi}{d\tau} = k \cdot u = \frac{k \cdot u_0}{1 - \tau_0 a_0^2 (k \cdot u_0)\psi(\xi)}$$

where:

τ particle's proper time

k^α wave four-vector

u_0^α initial four-velocity

$$\psi(\xi) = \int_0^\xi \left[\hat{A}'(y)\right]^2 dy$$

for linearly polarized plane wave: $$\psi(\xi) = -\frac{1}{2}(\xi + \frac{1}{2}\sin 2\xi)$$

this gives: $$\tau(\xi) = \frac{\xi}{k \cdot u_0} + \frac{1}{8} a_0^2 \tau_0 \left[1 + 2\xi^2 - \cos 2\xi\right]$$