Radiation and Acceleration

- The problem with linear electromagnetic theory
- The Landau-Lifshitz equation
- Solution of the Landau-Lifshitz equation
- Habs macro particles
- Future objectives

The Problem of linear E&M Theory

In linear electromagnetic theory a charge q moves according to the Lorentz force equation: $\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{\beta} \times \vec{B}\right)$

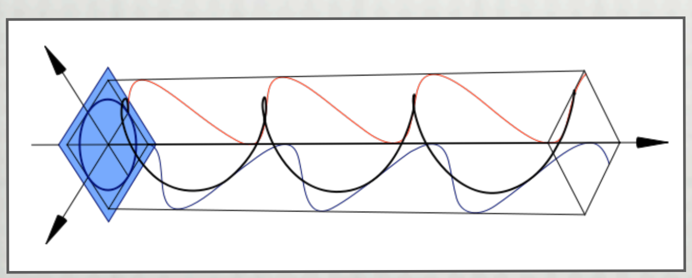
in covariant form: $m\dot{u}^{\alpha} = qF^{\alpha\beta}u_{\beta}$

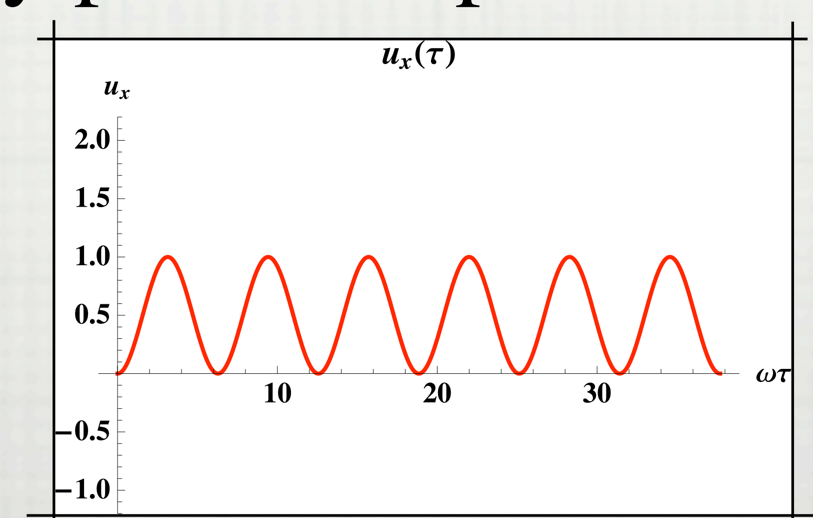
and radiates energy at a rate: $R = \frac{2}{3}q^2\dot{u}^{\alpha}\dot{u}_{\alpha}$

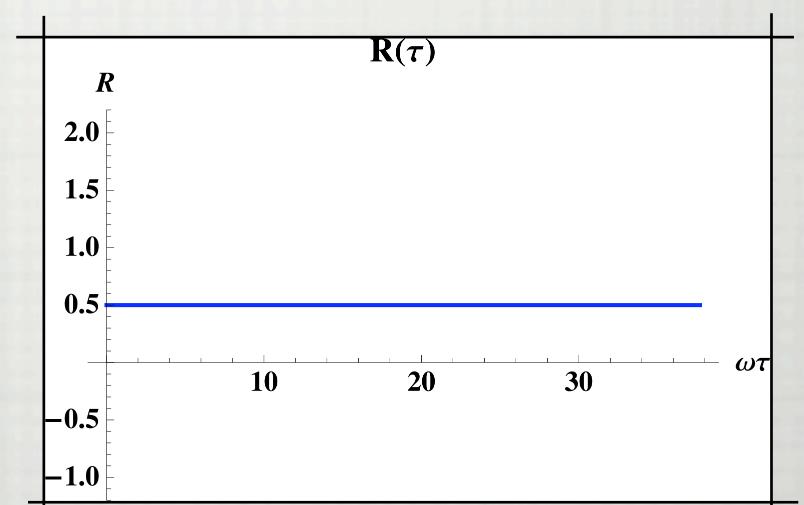
<u>Example: Circularly polarized plane wave</u>

$$\vec{A} = \frac{A_0}{\sqrt{2}}\left[\cos(\xi)\hat{y} - \sin(\xi)\hat{z}\right]$$

$$\xi = kx - \omega t$$







Lorentz force equation doesn't count for energy lost

The Landau-Lifshitz Equation

Adding the radiation-reaction effects to the Lorentz force gives Landau-Lifshitz equation:

$$m\dot{u}^{\alpha} = qF^{\alpha\beta}u_{\beta} + q\tau_0 \left[F^{\alpha\beta}_{,\gamma}u_{\beta}u^{\gamma} + \frac{q}{m}\left(\delta^{\alpha}_{\beta} + u^{\alpha}u_{\beta}\right)F^{\beta}_{\gamma}F^{\gamma}_{\delta}u^{\delta}\right]$$

where $\tau_0 = \frac{2q^2}{3m}$ and for a single electron: $\tau_0 \cong 6.24 \times 10^{-24}[\text{sec}]$

Many other equations were suggested (Lorentz-Abraham-Dirac, Caldirola, Mo-Papas and etc...)

Only Landau-Lifshitz equation is considered to be valid theoretically in the first order [Rohrlich 2002, Wald et al. 2009]

We can test it experimentally

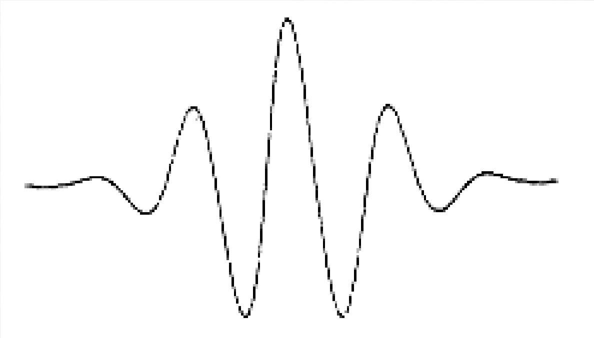
Towards a Solution of the Landau-Lifshitz Equation

Consider a charge in a transverse wave: $A^\alpha(x) = \mathtt{Re}\left[\varepsilon^\alpha f\left(\xi\right)\right]$

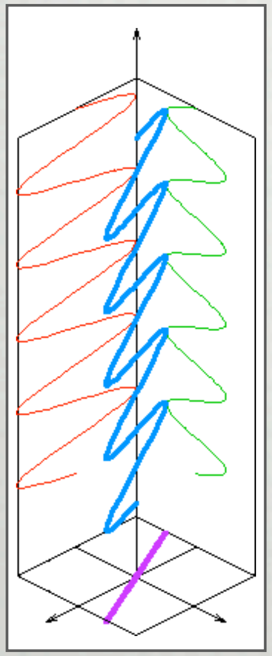
- Polarization vector ε^α
- Wave vector k^α
- $\xi = k \cdot x$

} Transverse condition: $k \cdot \varepsilon = 0$

Can still be a pulse:



Examples

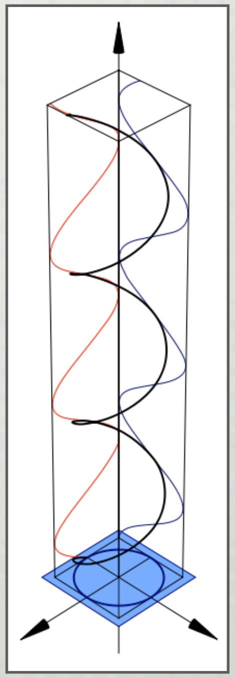


Linear Polarization

$$\begin{aligned} \varepsilon^\alpha &= (0,0,1,0) \\ k^\alpha &= (\omega, k, 0, 0) \\ f(\xi) &= A_0 \cos(\xi) \\ \vec{A} &= A_0 \cos(\xi)\hat{y} \end{aligned}$$

Circular Polarization

$$\begin{aligned} \varepsilon^\alpha &= \frac{1}{\sqrt{2}}(0,0,1,\pm i) \\ k^\alpha &= (\omega, k, 0, 0) \\ f(\xi) &= A_0 e^{i\xi} \\ \vec{A} &= \frac{A_0}{\sqrt{2}}\left[\cos(\xi)\hat{y} \mp \sin(\xi)\hat{z}\right] \end{aligned}$$



Solution of the Landau-Lifshitz Equation

$$
\begin{aligned}
u^\alpha &= \frac{k\cdot u}{k\cdot u_0}\overbrace{\left\{u_0^\alpha - \frac{q}{m}A_-^\alpha + \left[\frac{q}{m}u_0\cdot A_- - \frac{q^2}{2m^2}A_-^2\right]\frac{k^\alpha}{k\cdot u_0}\right\}}^{\text{Solution to Lorentz Eq.}} \\
&+ (k\cdot u)\tau_0\left\{\left[-\frac{q^3}{m^3}\phi^\alpha - \frac{q}{m}A_-'^\alpha\right] + \left[-\frac{q^4}{m^4}\phi\cdot A_- - \frac{q^2}{m^2}\psi - \frac{q^2}{m^2}A_-\cdot A_-' + \frac{q^3}{m^3}u_0\cdot\phi + \frac{q}{m}u_0\cdot A_-'\right]\frac{k^\alpha}{k\cdot u_0}\right\} \\
&+ (k\cdot u)\tau_0^2\left\{-\frac{q^2}{2m^2}\left(A_-' + \frac{q^2}{m^2}\phi\right)^2 - \frac{q^4}{2m^4}\psi^2\right\}k^\alpha
\end{aligned}
$$

with ψ, ϕ, $k\cdot u$ are functions of A' and τ_0

$$A_- = A(\xi) - A(0) \qquad A_-' = A'(\xi) - A'(0)$$

and ξ is an explicit function of τ

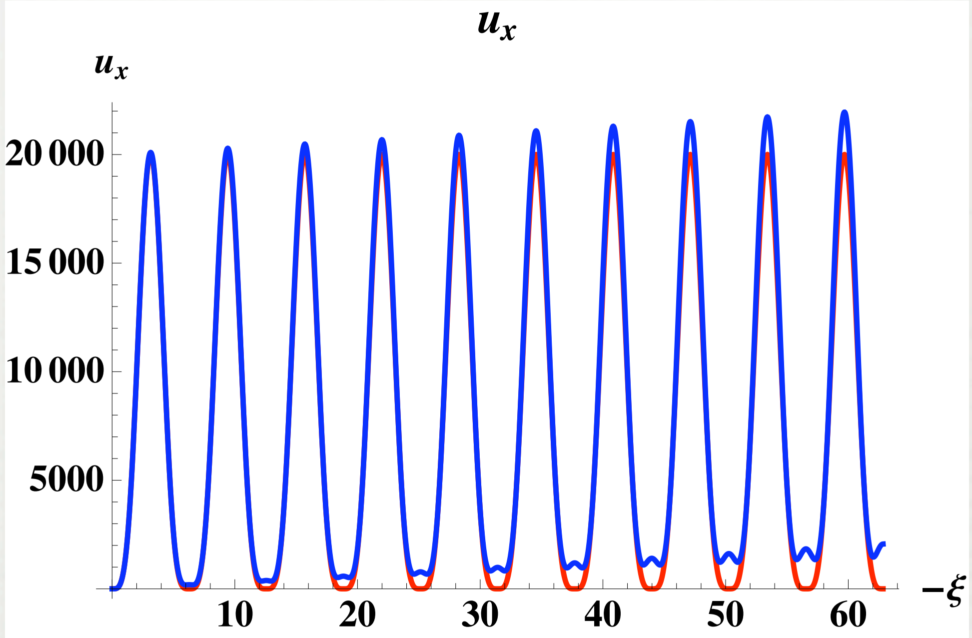
* The supplemental slides contain the explicit expressions.

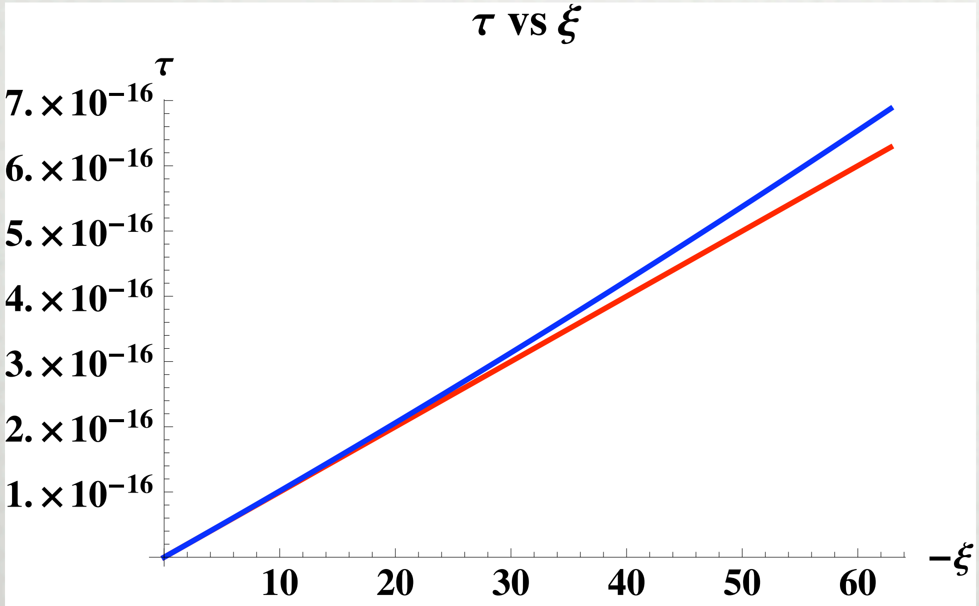
Comparison for linearly polarized plane wave for a single particle (n=1)

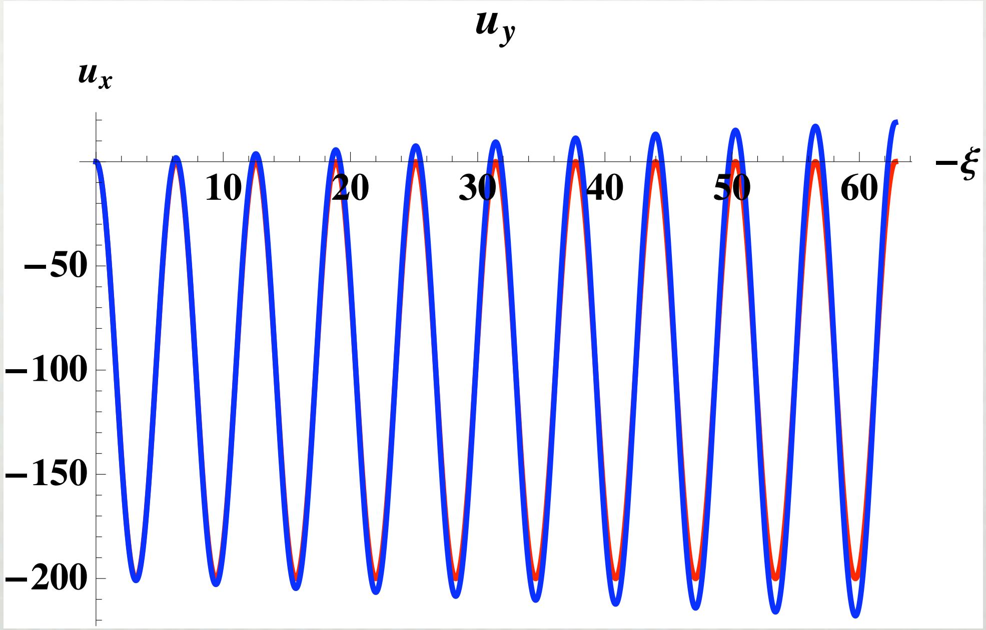
Lorentz Landau-Lifshitz

$$w = 10^{17}[\text{sec}^{-1}]$$
$$a_0 = 100$$
$$n = 1$$

$$\tau = -\frac{\xi}{\omega} + \frac{q^2}{m^2} a_0^2 \tau_0 \left(\frac{1}{4}\xi^2 + \frac{1}{8}\cos(2\xi) \right)$$



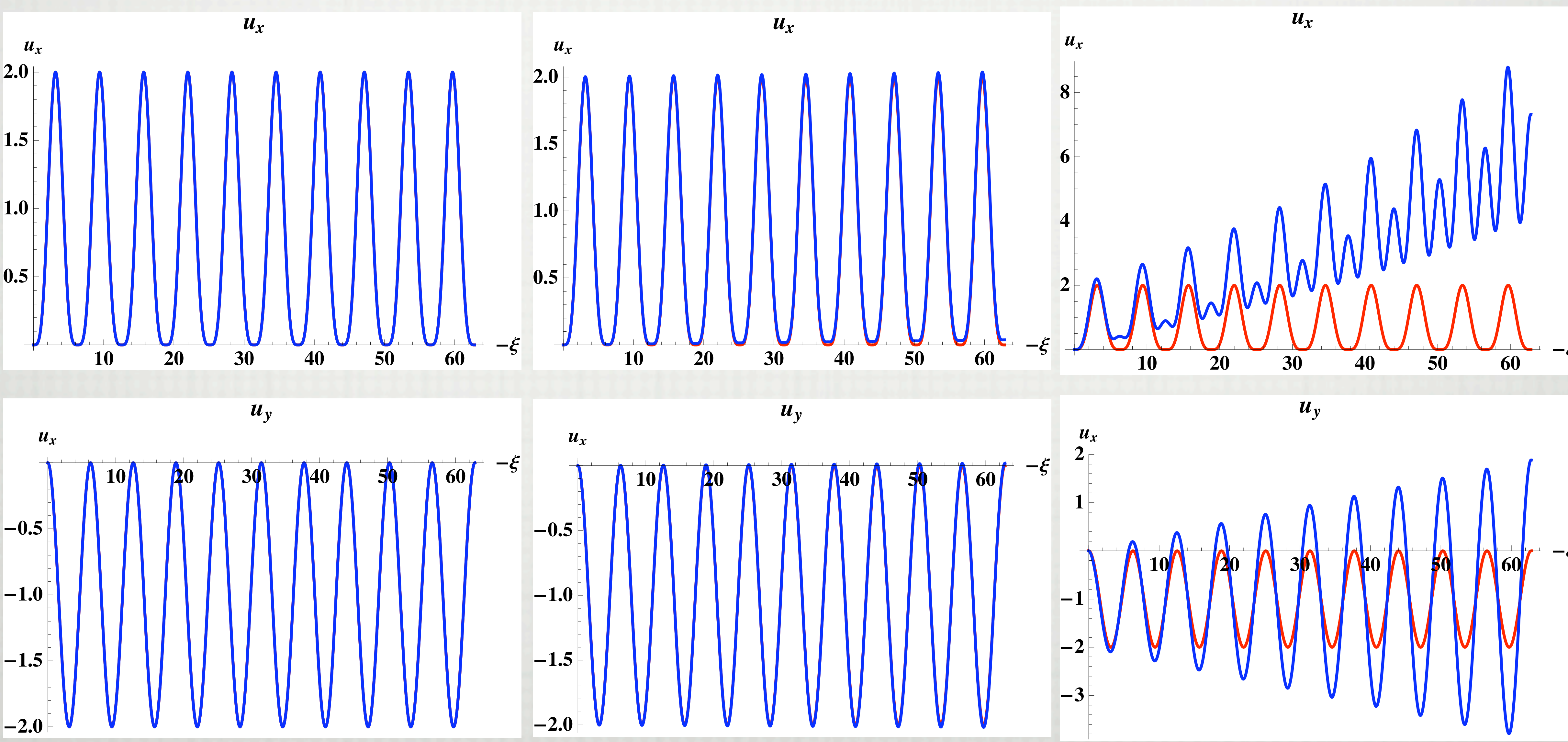




Comparison for a macro particle of n electrons

$a_0 = 1, w = 10^{15}[\text{sec}^{-1}]$ Lorentz Landau-Lifshitz

$n = 1$ $n = 10^5$ $n = 10^7$



Future Objectives

- Obtain complete solution for Landau-Lifshitz for ‘the realistic case’ (non-transverse 3D waves)
- Explore the transition from classical Landau-Lifshitz to a quantum theory
- Applications of new radiation effects

Credits

This talk is the result of many fruitful discussions and collaboration with:

- L. Labun & J. Rafelski
- N. Elkina & H. Ruhl
- P. Thirolf & D. Habs
- B. M. Hegelich

Thank you!

Supplement #1
Solution of the Lorentz Equation

Linear Polarization

$$
\begin{aligned}
\gamma(\tau) &= 1 + \frac{Q^2}{M^2}\frac{a_0^2}{2}\left[1-\cos(\omega\tau)\right]^2 \\
(\gamma v_x)(\tau) &= \frac{Q^2}{M^2}\frac{a_0^2}{2}\left[1-\cos(\omega\tau)\right]^2 \\
(\gamma v_y)(\tau) &= \frac{Q}{M}a_0\left[1-\cos(\omega\tau)\right] \\
(\gamma v_z)(\tau) &= 0
\end{aligned}
$$

Circular Polarization

$$
\begin{aligned}
\gamma(\tau) &= 1 + \frac{Q^2}{M^2}\frac{a_0^2}{2}\left[1-\cos(\omega\tau)\right] \\
(\gamma v_x)(\tau) &= \frac{Q^2}{M^2}\frac{a_0^2}{2}\left[1-\cos(\omega\tau)\right] \\
(\gamma v_y)(\tau) &= \frac{Q}{M}\frac{a_0}{\sqrt{2}}\left[1-\cos(\omega\tau)\right] \\
(\gamma v_z)(\tau) &= \mp\frac{Q}{M}\frac{a_0}{\sqrt{2}}\sin(\omega\tau)
\end{aligned}
$$

Supplement #2
Towards a Solution of Landau-Lifshitz Equation

Changing variables $\tau \to \xi = k \cdot x$ gives

$$k \cdot u = \frac{k \cdot u_0}{1 + \frac{q^2 \tau_0}{m^2}(k \cdot u_0)\psi(\xi)}$$

(compare with the Lorentz case: $k \cdot u = k \cdot u_0$)

Therefore,

$$\tau(\xi) = \frac{\xi}{k \cdot u_0} + \frac{q^2 \tau_0}{m^2} \int_0^\xi \psi(y) dy$$

with the structure integrals:

$$\psi(\xi) = \int_0^\xi \left[A'(y)\right]^2 dy \qquad \phi^\alpha(\xi) = \int_0^\xi A'^\alpha(y) \psi(y) dy$$