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"Acceleration and Radiation"

6-week progress report

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Outline

* Is there a problem?
* Lorentz-Abraham-Dirac (LAD) Equation
* Runaway & pre-acceleration solutions
* Other approaches
* Example: Model dependent physics predictions in oscillating electric field
* Towards a consistent picture: Radiation in Kaluza theory
* Future objectives

The Inconsistency in Lorentz Force

The Lorentz force $m\dot{u}^\alpha = eF^{\alpha\beta}u_\beta$

Larmor showed that the radiation rate of an accelerated charge is

$$\frac{dE_R}{dt} = \frac{2}{3}e^2\dot{v}^2$$

Relativistically (Heaviside-Abraham) the lost of energy-momentum due to radiation is

$$\dot{P}^\alpha = \frac{2}{3}e^2\left(\dot{u}^\beta\dot{u}_\beta\right)u^\alpha$$

Or using $0 = \frac{d}{d\tau}\left(\dot{u}^\beta u_\beta\right) = \ddot{u}^\beta u_\beta + \dot{u}^\beta\dot{u}_\beta$ we get

$$\dot{P}^\alpha = -\frac{2}{3}e^2\left(\ddot{u}^\beta u_\beta\right)u^\alpha$$

An accelerating charge loses energy! An effect that is missing in Lorentz force.

Lorentz-Abraham-Dirac equation
Derivation by Dirac

Dirac uses the retarded Liénard-Wiechert potential (solution of Maxwell equations), conservation of energy and "simplicity" to derive the covariant Lorentz-Abraham equation

$$m\dot{u}^{\alpha} = eF^{\alpha\beta}u_{\beta} + m\tau_0 \left(\delta^{\alpha}_{\beta} + u^{\alpha}u_{\beta}\right)\ddot{u}^{\beta}$$

"But whereas these equations, as derived from the Lorentz theory, are only approximate, we now see that <u>there is good reason for believing them to be exact, within the classical theory.</u>"

Dirac assumes the simplest possible form of energy flow, and justifies as follows:

"[Other forms] are all much more complicated than [the form used], so that one would hardly expect them to apply to a simple thing like an electron." Dirac, 1938

Runaway solutions

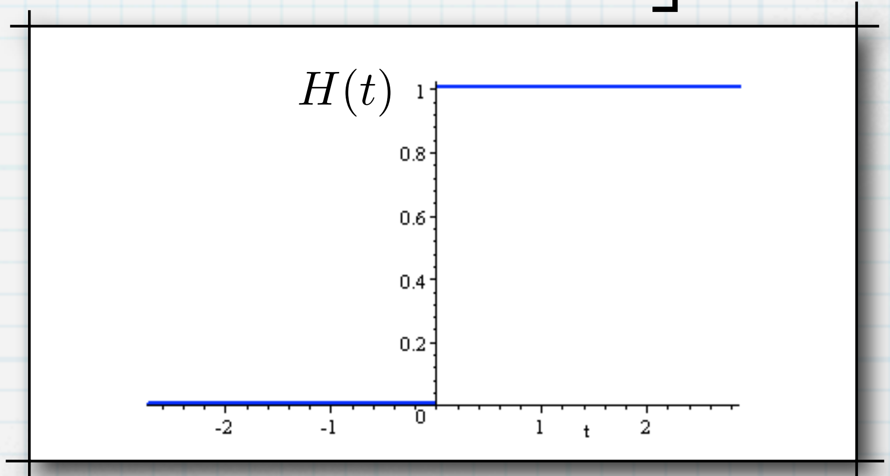
The non-relativistic LAD (F_L is the Lorentz force)

$$a - \tau_0 \dot{a} = \frac{1}{m} F_L$$

has a general solution

$$a\left(t\right) = e^{t/\tau_0}\left[a_0 - \frac{1}{m\tau_0}\int_0^t F_L\left(t'\right) e^{-t'/\tau_0} dt'\right]$$

for an electric field $H(t)E$ that is turned abruptly at t=0



$$a\left(t\right) = e^{t/\tau_0}\left[a_0 - \frac{eE}{m}\left(1 - e^{-t/\tau_0}\right) H\left(t\right)\right]$$

the acceleration grows exponentially!

Pre-acceleration solutions

We exclude runaway solutions by setting

$$a_0 = \frac{eE}{m}[H(t) + H(-t)]$$

and obtain

$$a(t) = \frac{eE}{m} e^{t/\tau_0} H(-t) + \frac{eE}{m} H(t)$$

the particle moves before time t=0.

So the effect precedes the cause...

The principle of causality breaks down!

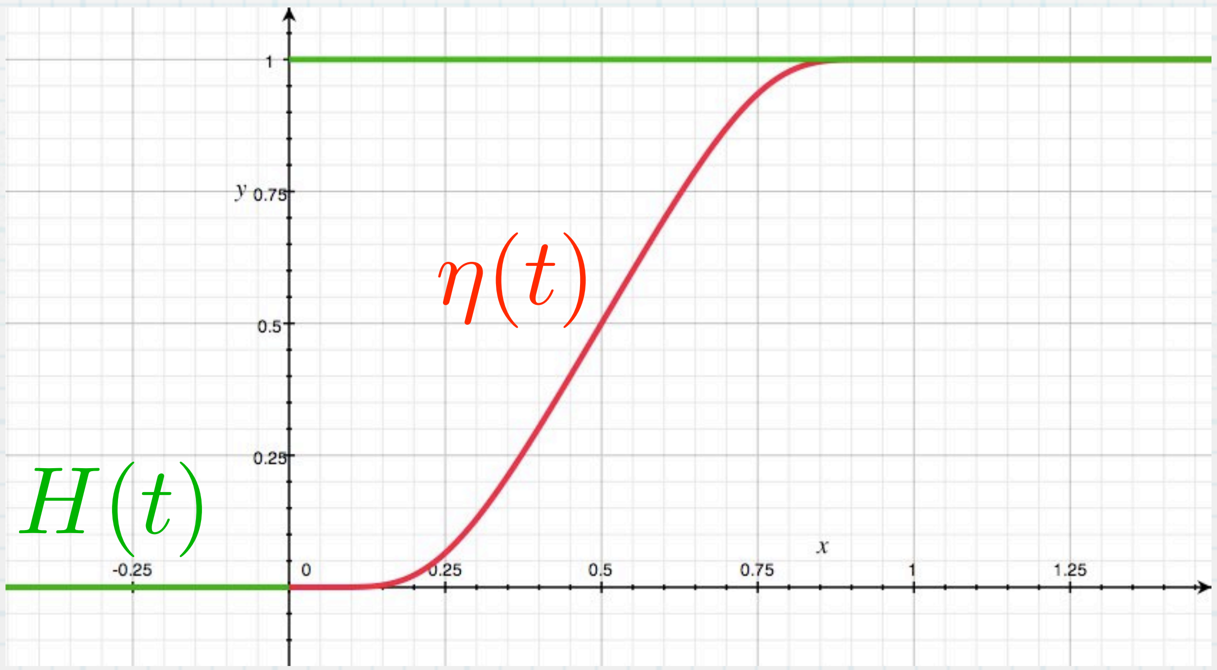
...does it?!

Resolving the Inconsistencies

The derivation of the Lorentz-Abraham-Dirac equation is based on a Taylor expansion of the field.

But a step function $H(t)E$ is non-analytic and cannot be expanded!

Solution: Take a smooth electric field $\eta(t)E$ that vanishes at t=0 and assume that the acceleration ceases as $t \to \infty$



This removes all pathologies (Yaghjian 1992).

Landau-Lifshitz equation

The LAD equation:

$$m\dot{u}^{\alpha} = eF^{\alpha\beta}u_{\beta} + m\tau_0\left(\delta^{\alpha}_{\beta} + u^{\alpha}u_{\beta}\right)\ddot{u}^{\beta}$$

If τ_c is a characteristic time scale, the last term is of order τ_0/τ_c

In the first order we get the Lorentz equation:

$$m\dot{u}^{\alpha} \cong eF^{\alpha\beta}u_{\beta}$$

Hence,

$$m\ddot{u}^{\alpha} \cong e\frac{d}{d\tau}(F^{\alpha\beta}u_{\beta})$$

Using this approximation in the LAD equation, gives the Landau-Lifshitz equation

$$m\dot{u}^{\alpha} = eF^{\alpha\beta}u_{\beta} + e\tau_0\left[F^{\alpha\beta}_{,\gamma}u_{\beta}u^{\gamma} + e/m\left(\delta^{\alpha}_{\beta} + u^{\alpha}u_{\beta}\right)F^{\beta}_{\gamma}F^{\gamma}_{\delta}u^{\delta}\right]$$

Mo-Papas equation

The Lorentz equation:

$$m\dot{u}^\alpha = eF^{\alpha\beta}u_\beta$$

The radiation-reaction force is (Larmor-Heaviside-Abraham)

$$F^\alpha_{RR} = -\frac{2}{3}e^2\dot{u}_\beta\dot{u}^\beta u^\alpha$$

In the first order approximation

$$F^\alpha_{RR} \cong -\frac{2}{3}\frac{e^3}{m}F^{\beta\gamma}\dot{u}_\beta u_\gamma u^\alpha$$

Postulate: The particle experiences another force $e\tau_0 F^{\alpha\beta}\dot{u}_\beta$

$$m\dot{u}^\alpha = eF^{\alpha\beta}u_\beta + e\tau_0\left(F^{\alpha\beta}\dot{u}_\beta + F^{\beta\gamma}\dot{u}_\beta u_\gamma u^\alpha\right)$$

Caldirola's equation

Iterations of the LAD give infinitely many derivatives of the velocity, hence a non-local theory.

The non-locality means that the four-velocity $u^\alpha(\tau)$ depends on the four-velocity at a previous time $u^\alpha(\tau - \Delta\tau)$

Postulate a universal quanta of time - the chronon τ_0

Want: A finite-difference relativistic equation for the four-velocity, that in the limit $\tau_0 \to 0$ gives the LAD

Get Caldirola's equation

$$\frac{m}{\tau_0}\left\{\cancel{u^\alpha(\tau)} - u^\alpha(\tau-\tau_0) + \underbrace{u^\alpha(\tau)\, u_\beta(\tau)\left[\cancel{u^\beta(\tau)}}_{-u^\alpha(\tau)} - u^\beta(\tau-\tau_0)\right]\right\} = eF^{\alpha\beta}(\tau)\, u_\beta(\tau)$$

Caldirola-Yaghjian equation

Postulate: the electron has the internal structure of a charged sphere

Compute the self-force due to its internal structure in the particle's rest frame

Lorentz transforming to a general frame, gives

$$m\dot{u}^{\alpha} = eF^{\alpha\beta}u_{\beta} + \underbrace{\frac{m}{\tau_0}\left[u^{\alpha}\left(\tau-\tau_0\right) + u^{\alpha}\left(\tau\right)u^{\beta}\left(\tau\right)u_{\beta}\left(\tau-\tau_0\right)\right]}_{\text{Caldirola term}}$$

The candidates

LAD (1904-1938)	$m\dot{u}^{\alpha} = eF^{\alpha\beta}u_{\beta} + m\tau_0 \left(\delta^{\alpha}_{\beta} + u^{\alpha}u_{\beta}\right)\ddot{u}^{\beta}$
Landau-Lifshitz (1951)	$m\dot{u}^{\alpha} = eF^{\alpha\beta}u_{\beta} + e\tau_0 \left[F^{\alpha\beta}_{,\gamma}u_{\beta}u^{\gamma} + e/m\left(\delta^{\alpha}_{\beta} + u^{\alpha}u_{\beta}\right)F^{\beta}_{\gamma}F^{\gamma}_{\delta}u^{\delta}\right]$
Mo-Papas (1971)	$m\dot{u}^{\alpha} = eF^{\alpha\beta}u_{\beta} + e\tau_0 \left(F^{\alpha\beta}\dot{u}_{\beta} + F^{\beta\gamma}\dot{u}_{\beta}u_{\gamma}u^{\alpha}\right)$
Caldirola (1979)	$-m/\tau_0 \left[u^{\alpha}(\tau-\tau_0) + u^{\alpha}(\tau)\, u_{\beta}(\tau)\, u^{\beta}(\tau-\tau_0)\right] = eF^{\alpha\beta}(\tau)\, u_{\beta}(\tau)$
Caldirola-Yaghjian (1992)	$m\dot{u}^{\alpha} = eF^{\alpha\beta}u_{\beta} + m/\tau_0 \left[u^{\alpha}(\tau-\tau_0) + u^{\alpha}(\tau)\, u^{\beta}(\tau)\, u_{\beta}(\tau-\tau_0)\right]$

Example: different physics in an oscillating electric field

Take $\vec{E} = E\cos(\omega t)\,\hat{x}$ (v denotes 3-velocity)

Lorentz and Mo-Papas eq.	$\dot{v} = \frac{eE}{m}\cos(\omega t)$
LAD	$\dot{v} = \frac{eE}{m}\cos(\omega t) - \frac{2}{3}\frac{e^2}{m}\frac{\dot{v}^2 v - \ddot{v}\left(1+v^2\right)}{\left(1+v^2\right)^{3/2}}$
Landau-Lifshitz	$\dot{v} = \frac{eE}{m}\cos(\omega t) + \frac{eE}{m}\tau_0\omega\sin(\omega t)\sqrt{1+v^2}$
Caldirola	$v\sqrt{1+v_-^2} - v_-\sqrt{1+v^2} = \frac{eE}{m}\tau_0\cos(\omega t)$
Caldirola-Yaghjian	$\dot{v} = \frac{eE}{m}\cos(\omega t) - \frac{1}{\tau_0}\left[v\sqrt{1+v_-^2} - v_-\sqrt{1+v^2}\right]$

Kaluza Theory

Consider a five-dimensional space-time, with the metric

$$\hat{g}_{AB} = \begin{bmatrix} g_{ab} + k^2\phi^2 A_a A_b & k\phi^2 A_a \\ k\phi^2 A_b & \phi^2 \end{bmatrix}$$

Variating the 5D Hilbert-Einstein action

$$S = -\frac{1}{16\pi\hat{G}} \int \hat{R}\sqrt{-\hat{g}}d^5x$$

Get a unification of Einstein's equation and Maxwell's equations:

$$\begin{aligned} G_{ab} &= \frac{k^2\phi^2}{2}T_{ab} - \frac{1}{\phi}\left(\nabla_a\left(\partial_b\phi\right) - g_{ab}\Box\phi\right) \\ \nabla^a F_{ab} &= -3\frac{\partial^a\phi}{\phi}F_{ab} \\ \Box\phi &= \frac{k^2\phi^3}{4}F_{ab}F^{ab} \end{aligned}$$

Back to radiation

By the equivalence principle, gravity and acceleration are intimately related.

Both gravity & electromagnetism enter the radiation effects. That's where the unification scheme of Kaluza comes into play.

The 4D geodesic equation from 5D Kaluza theory is

$$\underbrace{\dot{u}^a + \Gamma^a_{bc} u^b u^c}_{\text{Einstein term}} = \underbrace{\frac{e}{m} F^{ab} u_b}_{\text{Lorentz}} + \underbrace{G^a_{bc} u^b u^c}_{\text{Lifshitz}} + \underbrace{H^a_b u^b u^5 + I^a \left(u^5\right)^2}_{\text{Additional terms}}$$

where G, H and I are tensors that depend non-linearly on the fields.

The equation is very similar to the Landau-Lifshitz equation, has no pathological solutions but requires further exploration of the motion in the fifth dimension.

Future objectives

* Search for generalized action principle leading to existing models
* Search for an action principle unifying gravity and electromagnetism (example: Kaluza theory)
* Obtain τ_0^2 terms:

$$m\dot{u}^\alpha = eF^{\alpha\beta}u_\beta + \tau_0 X^\alpha + \tau_0^2 Y^\alpha + \ldots$$

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