NONLOCALITY: THE PRICE OF DETERMINISM & REALISM

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Abstract. Since the introduction of quantum theory there was a great unsatisfaction with its interpretation. Today, after nearly a century in which the theory has been tested and provided us with unpresidential agreement with experiments, quantum mechanics prevailed and is considered to be one of the cornerstones of modern physics. Yet the nondeterministic consequences of the measurement process and the collapse of the wave function in the Copenhagen interpretation are far from being pleasing or even well understood. This paper presents an alternative interpretation of quantum mechanics due to Bohm in which determinism is preserved, and quantum effects are the outcome of "hidden variables". It is shown by Bell's inequality that any deterministic hidden variable theory that produces the results of quantum mechanics (and in particular Bohm's) is nonlocal. We conclude with a discussion of the philosophical implications of Bell's inequality and Bohm's interpretation of quantum mechanics.

1. Introduction

The predictions of quantum mechanics agree very well with experimental data, and up to the present time there is no evidence for any domain in which quantum theory breaks down. Even its relativistic descendant quantum electrodynamics agrees marvelously with experiments and is considered to be the most stringently tested theory in the history of physics. Nevertheless, almost every one of us remained baffled when being first introduced to the theory of quantum mechanics. More remarkable is the fact that although the theory became the core of mainstream physics so rapidly, its founders were not fully content with its ontological implications neither. This unsatisfaction can be best demonstrated by the Nobel laureates Murray Gell-Mann and Max Von Laue, who said (respectively) "Quantum mechanics, that mysterious, confusing disciplines, which none of us really understands but which we know how to use"[7] and "If that turns out to be true, I'll quite physics".

In the last few decades we have witnessed the remnants of the famous Bohr-Einstein debate slowly decaying, as the great majority of physicists fully accept the Copenhagen interpretation of quantum mechanics. Whenever any doubts about quantum mechanics arise, it is often argued that quantum physics require a radical revision of our world view, as our intuition from the interaction with the macro-world cannot be expected to hold for the micro-world as well. It was Richard Feynman who said to his quantum mechanics students[6]: "That's the way nature works. You don't like it? Go somewhere else! To another universe where the rules are simpler and philosophically more pleasing...". I do not share this view at all.

Even if the predictions of quantum mechanics coincide with experiments, it does not mean that basic questions concerning its foundations should not be asked. On the contrary, the fact that quantum mechanics is so successful should encourage us to stop and (perpetually) rethink its philosophical implications on reality itself. Quantum mechanics provides us with a great tool for computing probabilities of experimental results, but gives no insight about reality per se.

Of course, whether or not a physical theory should provide a physical account of reality itself, and in what way it is even possible (or impossible) to describe reality itself is a complex matter that will not be discussed here. Instead, the goal of this paper is to demonstrate that there are alternative interpretations of quantum mechanics that are fully deterministic and accounting for all the phenomena described by nonrelativistic quantum mechanics. We will concentrate our attention on Bohmian mechanics [4, 5], although alternative theories exist (e.g. Stochastic Mechanics[8]) . We will see that the obscure nature of physical reality due to the measurement process, which is one of the most controversial issues in quantum mechanics turns out to be the most appealing aspect of Bohmian mechanics, in which reality receives an objective description independently of whether or not we observe it.

Since Bohmian mechanics is usually criticized as a valid substitution for the Copenhagen interpretation of quantum theory, we will discuss typical criticisms of it. The most solid line of attack on Bohmian mechanics is based on the fact that the theory is nonlocal, a feature that is considered to be an artifact of (prerelativistic) physical theories that is not a part of the physical world. This issue will be addressed by a short discussion of Bell's theorem[3], with which we demonstrate that any deterministic hidden variable theory that makes the same predictions as quantum mechanics is doomed to be nonlocal.

2. Bohmian Mechanics

We begin our discussion with the Bohmian theory for one-body systems. The generalization to many-body systems will be considered later. Mathematically, the generalization is straightforward, but we will see that the latter has far greater implications.

2.1. **One-body systems.** The Schrödinger equation for one-body with mass m is

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\triangle\psi+V\psi \tag{2.1}$$

where V is the classical potential, $\psi\left(t,\vec{x}\right)$ is the wave function of the particle and $\hbar$ is reduced Planck constant. Since the wave function is complex, it can be expressed in polar coordinates as

$$\psi\left(t,\vec{x}\right)=R\left(t,\vec{x}\right)e^{iS(t,\vec{x})/\hbar} \tag{2.2}$$

where R and S are real-valued functions. Recall that in the Copenhagen interpretation of quantum mechanics, the function

$$P\left(t,\vec{x}\right)=R^2\left(t,\vec{x}\right)$$

gives the probability density of finding the particle in position $\vec{x}$ at time t. Substituting Eq. 2.2 in the Schrödinger Eq. 2.1 and separating real and imaginary parts gives

$$\frac{\partial S}{\partial t}+\frac{\left(\nabla S\right)^2}{2m}+V-\frac{\hbar^2}{2m}\frac{\triangle R}{R}=0 \tag{2.3}$$

and

$$\frac{\partial P}{\partial t} + \nabla \cdot \left(P \frac{\nabla S}{m} \right) = 0 \tag{2.4}$$

In the classical limit $\hbar \to 0$, these equations have a very simple interpretation as Eq. 2.3 reduces to

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V = 0 \tag{2.5}$$

This is the celebrated Hamilton-Jacobi equation satisfied by the phase function $S(t, \vec{x})$. In analytical mechanics, the Hamilton-Jacobi equation represents a particle trajectory (evolving according to Newton's second law) with velocity

$$\vec{v} = \frac{\nabla S}{m} \tag{2.6}$$

that is normal to the wave fronts (surfaces of constant $S(t, \vec{x})$). It follows then that we can express Eq. 2.4 as

$$\frac{\partial P}{\partial t} + \nabla \cdot (P \vec{v}) = 0 \tag{2.7}$$

Notice however that this is nothing other than the continuity equation. This shows that Eq. 2.7can be regarded as a conservation law for the probability in an ensemble of such particles, all moving with trajectories that are normal to the same wave front with probability density $P(t, \vec{x})$.

This reveals that although we started with a quantum one-body system, it can be easily reinterpreted as an ensemble of <u>classical particles</u> evolving deterministically (in the limit $\hbar \to 0$). A natural question to ask is, whether we can extend this classical interpretation of Eqs. 2.3 & 2.4 when $\hbar \neq 0$.

Note that Eq. 2.3 only differs from the classical Hamilton-Jacobi Eq. 2.5 in the last term, which we define to be

$$Q(t, \vec{x}) = -\frac{\hbar^2}{2m} \frac{\triangle R}{R} \tag{2.8}$$

and can play the role of an additional potential. Thus we may rewrite Eq. 2.3 as

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0 \tag{2.9}$$

where V is now called the classical potential and Q is called the quantum potential. We will refer to Eq. 2.9 as the quantum Hamilton-Jacobi equation.

In this case Eq. 2.7 still expresses probability conservation, but for an ensemble of particles which satisfies the quantum Hamilton-Jacobi Eq. 2.9 instead of Eq. 2.5. This shows that the previous interpretation is still valid provided that the potential affecting the ensemble of particles is the total potential $V + Q$, superposed from the classical potential V and the new quantum potential Q. To summarize, the new interpretation has the following key features:

- The particle is <u>a particle</u> and has <u>well-defined position</u> $x(t)$ which varies <u>continuously and deterministically</u> according to the equation of motion

$$m \frac{d^2 x}{dt^2} = -\nabla V - \nabla Q \tag{2.10}$$

  where $-\nabla V$ is the classical force acting on the particle and $-\nabla Q$ is the quantum force.

- Consequently, the particle is always affected by the new quantum field $Q(t,\vec{x})$ (or alternatively by the wave function $\psi(t,\vec{x})$). The quantum field is defined by Eqs.2.2 and 2.8.
- The quantum field $\psi(t,\vec{x})$ satisfies the Schrödinger Eq. 2.1, and therefore changes <u>continuously and deterministically</u>.
- The particle's velocity is restricted to the form given by Eq. 2.6.
- A statistical ensemble of such particles (having the same quantum field ψ) has the probability density $P = R^2$.

We can therefore see that the combined system of a particle with its quantum field ψ is fully deterministic. The statistical interpretation is only relevant if we consider an ensemble of such particles as in classical statistical mechanics. Moreover, the classical limit of the theory is obtained directly and easily by 2.10 when the gradient of the quantum potential goes to zero (e.g. when $\hbar \to 0$).

2.2. **Many-body systems.** In the case of an n-body system, the wave function $\psi(t,\vec{x}_1,\dots,\vec{x}_n)$ satisfies the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\sum_{k=1}^{n}\frac{\hbar^2}{2m_k}\triangle_{\vec{x}_k}\psi + V\psi$$

where m_k is the mass of the k-th body. Writing again $\psi = Re^{iS/\hbar}$ and separating real and imaginary parts gives

$$\frac{\partial S}{\partial t} + \sum_{k=1}^{n}\frac{\left(\nabla_{\vec{x}_k}S\right)^2}{2m_k} + V + Q = 0 \tag{2.11}$$

and

$$\frac{\partial P}{\partial t} + \sum_{k=1}^{n}\nabla_{\vec{x}_k}\cdot\left(P\vec{v}_k\right) = 0 \tag{2.12}$$

where $P = |\psi|^2$,

$$\vec{v}_k = \frac{1}{m_k}\nabla_{\vec{x}_k}S$$

and

$$Q = \frac{-\sum_{k=1}^{n}\frac{\hbar^2}{2m_k}\nabla^2_{\vec{x}_k}R}{R} \tag{2.13}$$

is the n-body quantum potential. As in the case for the one-body system, Eq. 2.11 can be interpreted as the quantum Hamilton-Jacobi equation for n bodies traveling deterministically by the influence of the classical potential V and the quantum potential Q. Evidently, the above is a direct extension of the previous one-body system for n-body systems and the features mentioned for the one-body system hold in this case as well.

Incidentally, even if a measurement process is considered, it can be described as a many-body system that includes the experiment itself and the measuring apparatus, and therefore evolves deterministically as well. Because an analysis of such a measuring process in Bohmian mechanics is a delicate matter, we will not dwell on the matter and the reader is referred to [5].

There is one substantial difference between the n-body system and the one-body system. Eq. 2.13 shows that the quantum potential contains the amplitude R in both the numerator and the denominator, and consequently does not necessarily fall off with the distance. For the one-body system, this means that the particle is constantly affected by its environment (through the quantum force $-\nabla Q$). In the n-body system the quantum potential may have a similar dependence on the environment. However, in addition the n bodies can also be strongly coupled and interact at long distances through the quantum potential Q. This means that Bohm's theory is <u>nonlocal</u>, and that particles may interact at a distance.

2.3. **Discussion of Bohmian Mechanics.** The above discussion shows that quantum phenomena can be equally explained as an extension of the classical Hamilton-Jacobi theory. It might seem that returning to old classical notions is inadequate in understanding the micro-world, as quantum effects have many features utterly different from those in classical mechanics. However, a thorough inspection of Bohmian mechanics shows that Bohmian mechanics has striking new features.

First and as noted earlier, since the quantum field does not necessarily fall off with the distance, even remote features of the environment may affect the dynamics of the particle. Moreover, we see from Eq. 2.8 that the quantum potential is invariant under multiplication by a constant. This means that the effects of the quantum potential are independent of the intensity of the quantum field ψ. By contradistinction, classical waves which act mechanically usually produce effects that are proportional to the intensity of the wave. This results in very different dynamics than in classical mechanics, as even in empty space with no classical forces the quantum force may act on the particle. Namely, a particle in empty space need not travel uniformly in straight lines and the quantum field is thus guiding the particle in its motion.

To demonstrate this, we may consider the perplexing results of the double slit experiment. In conventional quantum mechanics the results of the double-slit experiments are usually attributed to the (obscure) wave-particle duality. In Bohmian mechanics the interpretation is completely different as each particle is indeed a particle and has a well-defined position. In this case, the wave function (and thus the quantum potential) is traveling through both slits similarly to the original Young's double-slit experiment with water waves. Each particle has a well-defined trajectory and passes through exactly one slit. Bohmian mechanics is deterministic, so that the final position of each particle is completely determined by its initial position and initial velocity through Eq. 2.10. Because the initial position of each particle is not controllable by the experimenter, each particle ends up in a different position on the screen, resulting in the usual interference patterns on the detector's screen (see figure 2.1). This interference patterns for the particles are just the manifestation of the fact that the quantum field travels as a wave and is guiding the particles in their motion. This makes the explanation of the double-slit experiment rather intuitive.

3. Bell's Theorem

Bohmian mechanics (and other deterministic interpretations of quantum mechanics) are not taught as part of the core physics education in most universities. Consequently (and unfortunately) most physicists don't really know much about Bohmian mecanics. The few physicists who do know or heard of Bohmian mechanics usually criticize it by saying that it conflicts with either Bell's theorem or with the Copenhagen interpretation, and was therefore proven to be false. However, the latter argument is not valid since we do not have an independent proof that the

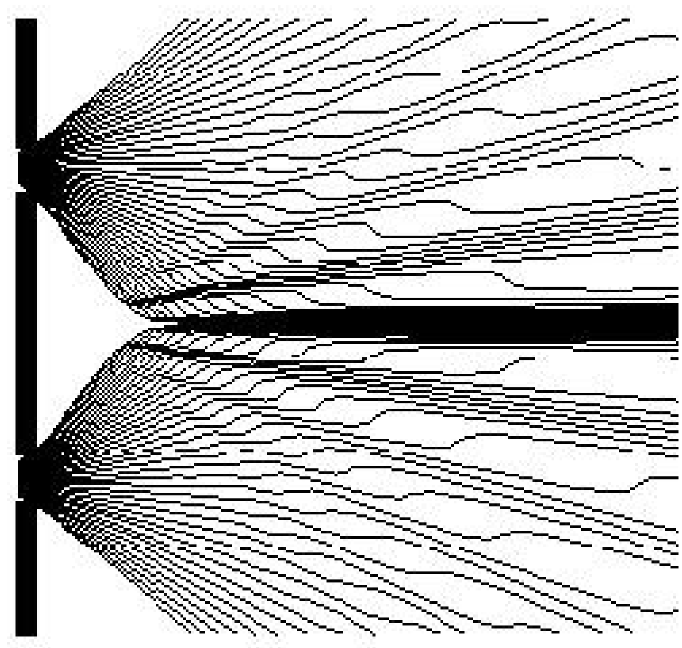

FIGURE 2.1. Trajectories of particles according to Bohmian mechanics in the double slit experiment

Copenhagen interpretation is necessarily true. The first argument is also incorrect, but requires a basic knowledge of Bell's theorem, which is the subject of the following section.

3.1. **The Einstein-Rosen-Podolsky Thought Experiment**[2]. As is well known, Einstein was never content with the Copenhagen interpretation of quantum mechanics. In a series of public debates[10] between Albert Einstein and Niels Bohr, Einstein attempted to contradict the Copenhagen interpretation on both physical, mathematical, and philosophical grounds. The debates culminated in a thought experiment suggested by Einstein, Rosen and Podolsky that puts to question the consistency of quantum mechanics and the principle of locality.

Consider a pair of free spin $\frac{1}{2}$ particles moving in opposite directions. If the spin of each particle is $\vec{\sigma}_i$ $(i = 1, 2)$, Stern-Gerlach magnets can be used to measure the component $\vec{\sigma}_i \cdot \vec{a}$ of each spin, where $\vec{a}$ is a given unit vector. If the measurement of the component $\vec{\sigma}_1 \cdot \vec{a}$ is 1, quantum mechanics' spin conservation gives that the measurement of $\vec{\sigma}_2 \cdot \vec{a}$ must be -1 (and vice versa). We assume that prior to the measurement process, enough time has passed so that the two particles are spatially separated and the two measurements (made at two distant locations) do not influence each other. Since we can predict the result of measuring any component of $\vec{\sigma}_2$ by previously measuring the same component of $\vec{\sigma}_1$, the result of any such measurement can be said to be predetermined.

But if the result of one measurement affects the result of another measurement made on a distant object, doesn't it violate the principle of locality? This is the essence of the Einstein-Rosen-Podolsky (EPR) paradox, with which Einstein, Rosen and Podolsky (REF) criticized the Copenhagen interpretation of quantum mechanics.

The EPR paradox is the most prominent thought experiment to challenge quantum mechanics, and its implications where outreaching. It was the motivation for the later work by J. S. Bell, who proved that no local deterministic hidden variable theory can produce the same results as quantum mechanics for the EPR experiment.

3.2. **Bell's Inequality** [3]**.** We assume that there exist a local hidden variable theory that can describe the EPR configuration. This means that quantum mechanics is incomplete and that other elements that are beyond contemporary technology influence quantum phenomena as we observe in nature. Mathematically, this means that there exists a (hidden) parameter λ affecting the result of the EPR experiment. The result A of measuring $\vec{\sigma}_1 \cdot \vec{a}$ is determined by $\vec{a}$ and λ, while the result B of measuring $\vec{\sigma}_2 \cdot \vec{b}$ is determined by $\vec{b}$ and λ, and for a spin $\frac{1}{2}$ particle we know that

$$\begin{aligned} A\left(\vec{a},\lambda\right) &= \pm 1 \\ B\left(\vec{b},\lambda\right) &= \pm 1 \end{aligned} \tag{3.1}$$

The crucial assumption we will make is that nature is local and the result B for the second particle does not depend on $\vec{a}$ (and vice versa, A is independent of $\vec{b}$). On an ensemble of such EPR experiments, we can use probability theory to describe the expected values of the measurements of the spin components. Let $\rho\left(\lambda\right)$ be the probability density function of the hidden parameter λ, thus the expectation value of the two components $\vec{\sigma}_1 \cdot \vec{a}$ and $\vec{\sigma}_2 \cdot \vec{b}$ is

$$P\left(\vec{a},\vec{b}\right) = \int d\lambda \rho\left(\lambda\right) A\left(\vec{a},\lambda\right) B\left(\vec{b},\lambda\right) \tag{3.2}$$

Theorem. *(Bell) Using the above notations, if*

1. *The measurements of $\vec{\sigma}_1$ don't influence the measurements of $\vec{\sigma}_2$ and vice versa.*
2. *There is a hidden variable λ that determine the result of the measurement as in 3.1, then for any three constant vectors $\vec{a},\vec{b},\vec{c} \in \mathbb{S}^3$, the following inequality holds:*

$$1 + P\left(\vec{b},\vec{c}\right) \geq \left|P\left(\vec{a},\vec{b}\right) - P\left(\vec{a},\vec{c}\right)\right| \tag{3.3}$$

Remark. Although we use a single-valued parameter λ, it can represent both a single variable or a set of variables which may be either discrete or continuous.

Proof. Since $\rho\left(\lambda\right)$ is a probability density and is therefore normalized,

$$\int d\lambda \rho\left(\lambda\right) = 1$$

and Eq. 3.2 together with Eqs. 3.1implies that

$$\left|P\left(\vec{a},\vec{b}\right)\right| \leq 1$$

Notice that since a measurement of the $\vec{\sigma}_1 \cdot \vec{a}$ component of the spin of the first particle gives an opposite measurement on the $\vec{\sigma}_2 \cdot \vec{a}$ component of the second particle, we have

$$A\left(\vec{a},\lambda\right) = -B\left(\vec{a},\lambda\right)$$

and therefore we can rewrite Eq. 3.2 as

$$P\left(\vec{a},\vec{b}\right) = -\int d\lambda \rho(\lambda) A(\vec{a},\lambda) A\left(\vec{b},\lambda\right)$$

If $\vec{c}$ is another unit vector then

$$\begin{aligned} P\left(\vec{a},\vec{b}\right) - P(\vec{a},\vec{c}) &= -\int d\lambda\rho(\lambda)\left[A(\vec{a},\lambda)A\left(\vec{b},\lambda\right) - A(\vec{a},\lambda)A(\vec{c},\lambda)\right] \\ &= \int d\lambda\rho(\lambda)\left[A^2\left(\vec{b},\lambda\right)A(\vec{a},\lambda)A(\vec{c},\lambda) - A(\vec{a},\lambda)A\left(\vec{b},\lambda\right)\right] \\ &= \int d\lambda\rho(\lambda)A(\vec{a},\lambda)A\left(\vec{b},\lambda\right)\left[A\left(\vec{b},\lambda\right)A(\vec{c},\lambda) - 1\right] \end{aligned}$$

where in the second line we used $A^2\left(\vec{b},\lambda\right) = 1$. Hence

$$\left|P\left(\vec{a},\vec{b}\right) - P(\vec{a},\vec{c})\right| \leq \int d\lambda\rho(\lambda)\left[1 - A\left(\vec{b},\lambda\right)A(\vec{c},\lambda)\right]$$

notice however that by definition the second term in the integrand integrates to $P\left(\vec{b},\vec{c}\right)$, therefore we have

$$\left|P\left(\vec{a},\vec{b}\right) - P(\vec{a},\vec{c})\right| \leq 1 + P\left(\vec{b},\vec{c}\right)$$

□

Remark. Bell type inequalities were also proven in more general cases (and in other variants). In particular, there are (essentially) equivalent results for stochastic hidden variable theories (like Nelson's theory [8]).

3.3. **Consequences of Bell's theorem.** Quantum mechanics predicts that

$$P\left(\vec{a},\vec{b}\right) = -\vec{a}\cdot\vec{b}$$

So if, for example, $\vec{a}$, $\vec{b}$ and $\vec{c}$ are measured with angles 0, θ and 2θ from a given axis respectively (on the same plane), Bell's inequality reduces to

$$|\cos\theta - \cos 2\theta| \leq 1 - \cos\theta$$

But for small values of θ the leading term on the left-hand-side is $3\theta^2/2$ while the right-hand-side is of order $\theta^2/2$. Therefore Bell's inequality does not hold, and the predictions of quantum mechanics contradict Bell's inequality. It can be further shown[3] by a generalization of Bell's inequality that the quantum mechanical expectation value cannot be even represented arbitrarily close in the form of Eq. 3.2.

It was argued that Bell's theorem shows that deterministic hidden variable interpretations of quantum mechanics do not exist, but this is not exactly the case. John Bell was a big follower of Einstein's in his view of quantum mechanics, and had philosophical difficulties accepting the Copenhagen interpretation of quantum mechanics. Bell's theorem then, is supposed to show that determinism and more importantly objective realism (the existence of physical reality independently of the observer), do not necessarily have to be forfeited. Instead, Bell's theorem shows that both determinism and objective realism may be preserved, but only for the price of abandoning the principle of locality.

Bell's inequality 3.3 was tested in several experiments. The experiments used polarized photos (for which there is an equivalent inequality) instead of fermions. The most thorough set of experiments was performed by Aspect et al. [1] and maintained the detection events outside of each other's light cones. The experiments found that Bell's inequality was violated. Considering the assumptions of Bell's inequality, this means that we have an experimental proof that if there are hidden variables, nature is nonlocal.

The reader should be aware that the experimental results concerning Bell's inequality should be taken with a pinch of salt, as the apparatus and results of Aspect's experiments were criticised by some physicists [9]. Up to the present time, no uncontroversial experiments proving that Bell's inequality breaks down were performed. Nevertheless, there is a strong reason to belief that the result still holds.

4. Conclusions

Quantum mechanics provoked intense debate about the nature of physical reality and the role of an observer in its interpretation. So far, although the theory remains invincible when tested in the laboratory, the most crucial assumption in quantum theory has never been tested experimentally. Namely, the assumption that a physical system is completely specified by a wave function, where the latter only determines the probabilities of physical measurements obtained in statistical ensemble of similar experiments.

Most physicists often say that objections to the Copenhagen interpretation of quantum theory are irrelevant, as it is in excellent agreement with experiments and no consistent alternative interpretations exist. We saw that Bohmian mechanics provides a consistent alternative to quantum theory, where physical reality obtains an objective description similarly to the one given in classical physics. In Bohmian mechanics the fuzzy wave-particle duality is dismissed, and particles evolve in a deterministic fashion according to a law analogous to Newton's second law. The particles are affected by a new field, the quantum field. The new quantum field does not drops off with the distance, and consequently the theory is inherently nonlocal.

Although conceptually, nonlocal 'action-at-a-distance' effects are difficult to accept on relativistic grounds, we showed by Bell's theorem that any deterministic hidden variables theory that produces the same results as quantum mechanics is necessarily nonlocal. Accordingly, Bell's theorem can be viewed as a limitation on the kind of physical theory we can have to describe nature as we know it. In this sense, it implies that locality, determinism and objective realism cannot coexist.

Since Bohmian mechanics was derived from the Schrödinger equation, the results of Bohmian mechanics and quantum mechanics are indistinguishable experimentally. Personally, this is the main criticism I have for Bohmian mechanics, as it is only an interpretation of quantum mechanics using (mostly) familiar concepts. Bohmian mechanics gives no account for the origin (either conceptually or mathematically) of the wave function, although I hope that a better understanding of it indeed exists. Since the quantum potential (and consequently all quantum effects) depends on the Planck constant $\hbar$, and the Planck constant can be constructed using the electric charge e, the electromagnetic constant K and the speed of light c, it seems that if the quantum potential is the result of a more fundamental process, such a process should be of an electromagnetic origin.

Of course any discussion of quantum mechanics is incomplete without a full description of the measuring process, and this paper is no exception. I hope to include in future versions of this paper a thorough study of the measurement problem, the measuring process in Bohmian mechanics and its stochastic variants, and what personally I find more intriguing, the relation between nonlocality and the problem of radiation-reaction.

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