SUPPLEMENT: ANOTHER APPROACH FOR PARTIAL FRACTIONS

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In class we integrated the function $\int \frac{10x-2x^2}{(x-1)^2(x+3)}dx$ by using partial fraction decomposition. Here we'll discuss a different method for finding the coefficients. The same method is described in the textbook (see section 7.4).

We decompose the integrand to:

$$\frac{10x-2x^2}{(x-1)^2(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3}$$

so far nothing has changed. In class we judiciously chose values for x to obtain the coefficients A, B, C easily. Here, instead, we will multiply by the denominator of the original function, namely $(x-1)^2(x+3)$. This gives

$$10x - 2x^2 = A(x-1)(x+3) + B(x+3) + C(x-1)^2$$

Foiling the parentheses on the right-hand-side and collecting powers of x yields

$$-2x^2 + 10x = (A+C)x^2 + (2A+B-2C)x + (-3A+3B+C)$$

since this equality must hold for all x (why?) the coefficients of each power of x on the left-hand-side and on the right-hand-side must be the same (namely, if -2 multiply x^2 on the left-hand-side, it must also multiply x^2 on the right-hand-side, so $A+C$ is -2). Therefore

$$\begin{aligned} A + C &= -2 \\ 2A + B - 2C &= 10 \\ -3A + 3B + C &= 0 \end{aligned}$$

This is a set of three linear equations with three unknowns. You can solve it and obtain $A = 1, B = 2, C = -3$. This is the same result as we got in class, obtained in a different way. Then the integration itself is easy. This method always works (as long as you used the correct decomposition...)

So why didn't I teach this method in class? Well, you probably noticed that solving the three equations can be quite tedious. Esepcially if you compare it to what we did in class. This method is indeed more straight-forward in the sense that it doesn't require any thinking. However, if you have a polynomial of order five in the denominator you'll have to solve five different equations! So you may choose the method you prefer in your homework and exams, but I suggest trying to solve a problem using both methods before you decide.